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ECE

PM 1 (B)

ACE Academy

Signals & Sytems → Part - 2

All the Best

Ch - 4 :- Fourier Transform (F.T.)

⇒ Transformation is the process in which one domain is converted to another domain such that signal analysis becomes easy.

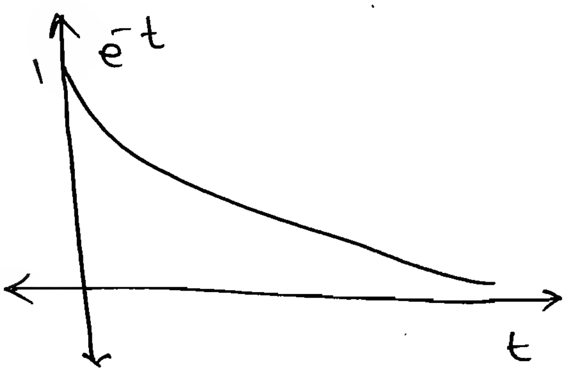
⇒ For any Non Periodic signal as $T \to \infty$ implies $\omega_0 \to 0$.

⇒ The discrete spectrum of Fourier series is converted to continuous spectrum in Fourier Transform.

⇒ Extension of F.S. is F.T.

⇒ F.T. is extension of F.S. to non-periodic signal.

⇒ $x(t) = e^{-t} \cdot u(t)$, $u(t)$



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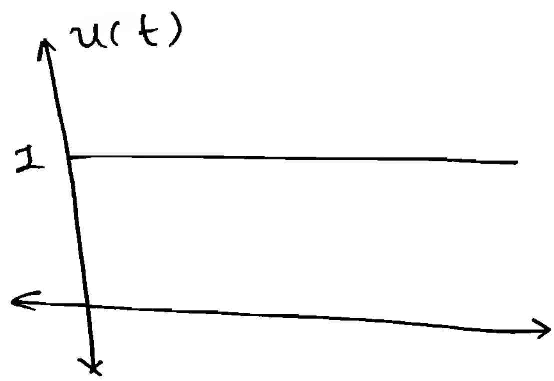
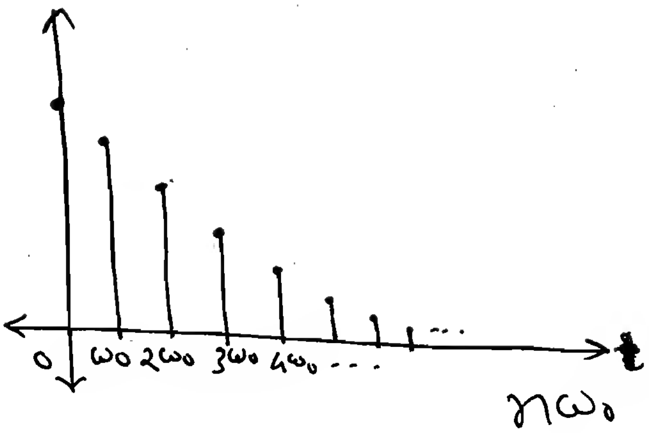


$T \to \infty$, Non-periodic.

$\Rightarrow$

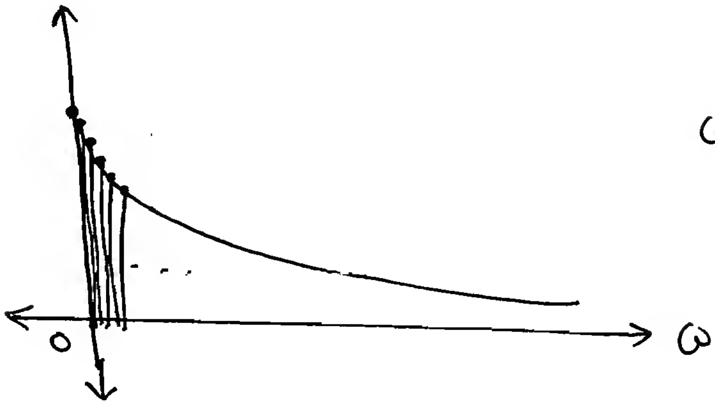


$T \to \infty$

$\omega \to 0$

i.e. $n\omega_0 \to \omega$.

$\Downarrow$



$\omega = n\omega_0$

$\Rightarrow$ From F.S.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{j\omega_0 n t}$$

&

$$C_n = \frac{1}{T} \int_{-T/2}^{+T/2} x(t) \cdot e^{-j\omega_0 n t} \cdot dt$$

$\boxed{T \to \infty}$

$$\lim_{T \to \infty} T\, C_n = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} \cdot dt$$

$$\Rightarrow \boxed{X(\omega) = \int_{-\infty}^{+\infty} x(t) \cdot \underbrace{e^{-j\omega t}}_{\text{kernel}} \cdot dt}$$

$\Downarrow$

F.T. of $x(t)$.

$\Rightarrow$ I.F.T.

$$\therefore \boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega).\, e^{j\omega t}.\, d\omega.}$$

$$\omega = 2\pi f$$

$$\therefore \; d\omega = 2\pi\, df.$$

I.F.T.

$$\therefore \boxed{x(t) = \int_{-\infty}^{\infty} X(f).\, e^{j2\pi f t}.\, df.}$$

$\Rightarrow$ F.T.

$$\boxed{X(f) = \int_{-\infty}^{+\infty} x(t).\, e^{-j2\pi f t}.\, dt}$$

$$\Rightarrow \boxed{2\pi\delta(\omega) = \delta(f).}$$

Proof:

$$2\pi\,\delta(\omega) = 2\pi\delta(2\pi f).$$

$$= \frac{\cancel{2\pi}}{\cancel{|2\pi|}}.\, \delta(f)$$

$$\boxed{2\pi\delta(\omega) = \delta(f).}$$

* Convergence of F.T.:

$\Rightarrow$ $X(\omega) < \infty$.

$\Rightarrow$ Fourier transform is defined for stable and energy signal. ✓

⇒ Fourier transform of a power signals is defined as approximation to energy signals (or) impulse functions are permitted.

⇒ Fourier transform is not defined for neither absolutely integrable nor square integrable signals.

$\boxed{P\ 4.1.1.}$ If $x(t)$ is a voltage waveform, then what are the units of $X(f)$?

Soln:

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t} \cdot dt$$

$= \text{Volts} \cdot \text{sec}.$

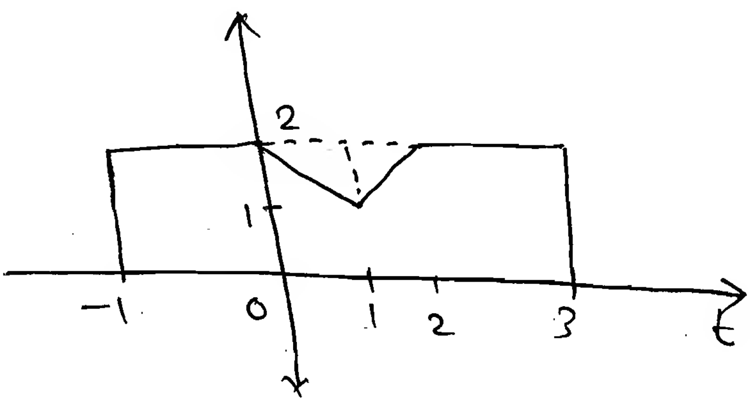
So, $X(f)$ unit is Volts. sec (or)

$\boxed{\text{Volts} / \text{Hz}}$

$\boxed{P\ 4.1.2}$ For the signal $x(t)$ shown in figure, find

(a) $X(0)$.

(b) $\int_{-\infty}^{+\infty} X(\omega)\, d\omega$



Sol^n:

(a) $$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \cdot dt.$$

make $\omega = 0$.

$$\therefore \quad X(0) = \int_{-\infty}^{+\infty} x(t) \cdot dt$$ = Area under the curve.

$$\therefore \quad X(0) = (4 \times 2) - \left(\tfrac{1}{2} \times 1 \times 2\right)$$

$$\boxed{X(0) = 7}$$

(b) $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) \cdot e^{j\omega t} \cdot d\omega.$$

make $t = 0$.

$$\therefore \quad x(0) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) \cdot e^{0} \cdot d\omega.$$

$$\Rightarrow \int_{-\infty}^{+\infty} X(\omega) \cdot d\omega = 2\pi x(0).$$

$$= 2\pi \times 2$$

$$= 4\pi.$$

Note:

→ Area under one domain corresponds to observing the other domain at origin.

$\boxed{P\ 4.1.3.}$ Consider the signal $x(t) = \begin{cases} e^{-t} ; t \geq 0 \\ 0 ; t < 0 \end{cases}$

and $X(\omega)$ is the F.T. of this signal.

Then the Value of $\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\, d\omega$ is ____.

Solⁿ:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) \cdot e^{j\omega t} \cdot d\omega$$

So, $x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, d\omega = e^{-0} = 1.$

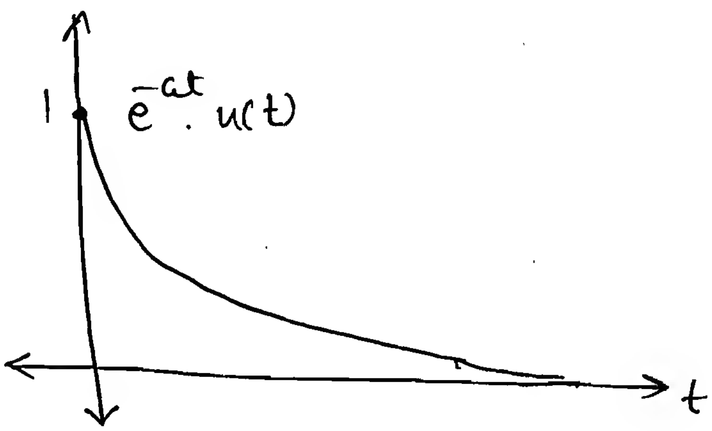
So, $$\boxed{\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) . d\omega = 1.}$$

* F.T. of Standard signals:

1) Decaying exponential

$x_1(t) = e^{-at} \cdot u(t)$, $a > 0$.

$\Rightarrow$



$$\therefore \quad X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \cdot dt.$$

$$= \int_{-\infty}^{+\infty} e^{-at} \cdot e^{-j\omega t} \cdot dt.$$

$$\therefore \quad X(\omega) = \int_{0}^{\infty} e^{-(a+j\omega)t} \cdot dt.$$

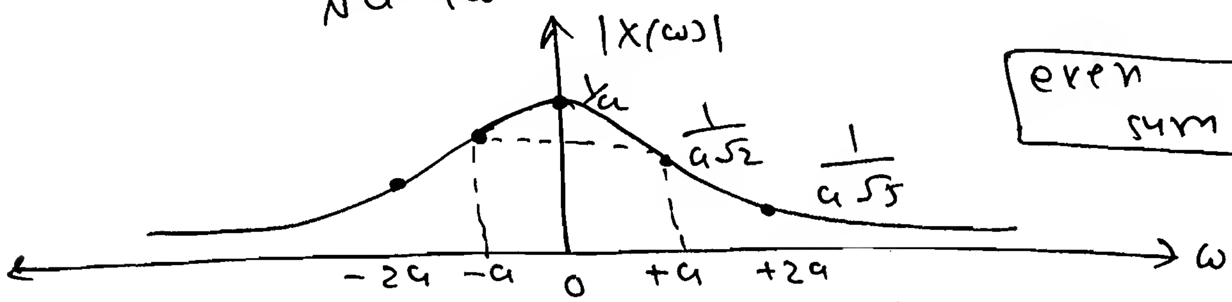
$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

$$X(\omega) = 0 + \frac{1}{a+j\omega}.$$

$$\therefore \quad \boxed{e^{-at} \cdot u(t) \xleftrightarrow{F.T.} \frac{1}{a+j\omega}.}$$

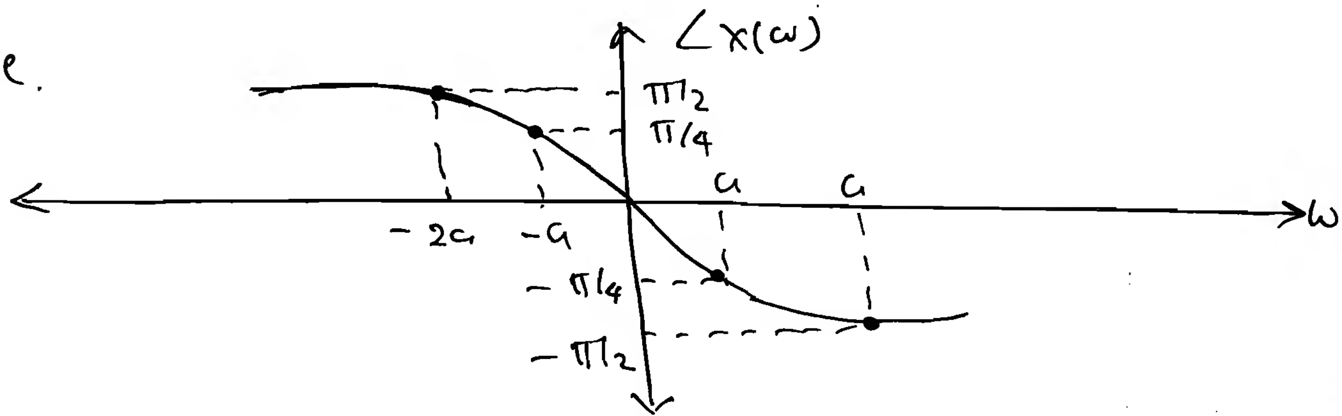
$$\Rightarrow \quad |X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}, \quad \angle X(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right).$$

Mag.



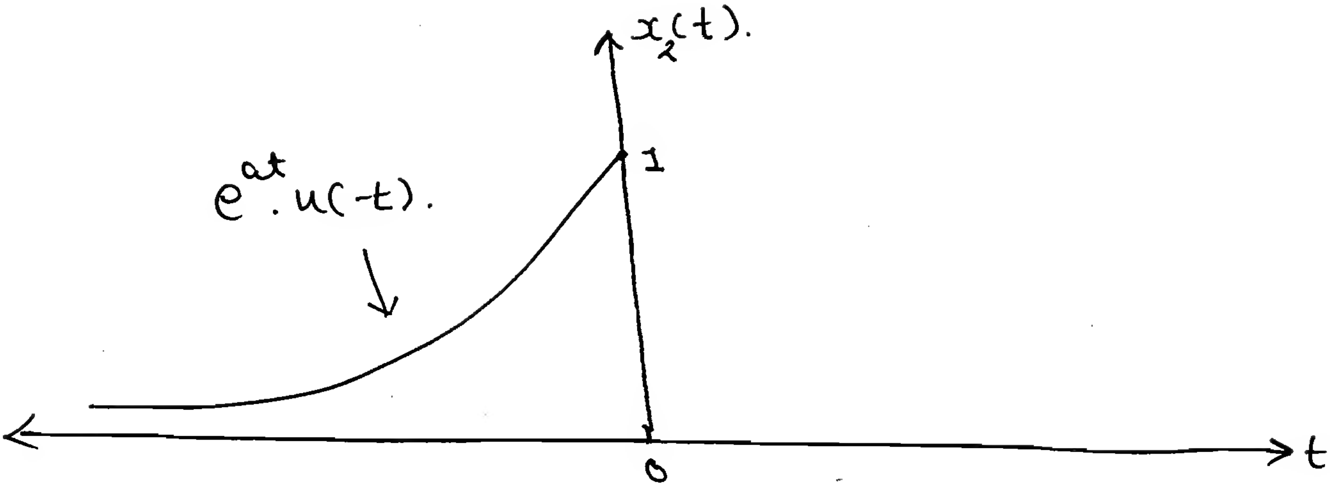
even sym.

Phase.



2) $e^{at} \cdot u(-t), \quad a>0.$

$\Rightarrow$



$\therefore \quad x_2(t) = x_1(-t)$

$\Rightarrow$ using time reversal property,

$$x(t) \xleftrightarrow{F.T.} X(\omega).$$

$$x(-t) \xleftrightarrow{F.T.} X(-\omega).$$

$$\therefore \quad X(-\omega) = \frac{1}{a - j\omega}.$$

$$\therefore \quad \boxed{e^{at} u(-t) \xleftrightarrow{F.T.} \frac{1}{a-j\omega}}.$$

(3) $x(t) = \delta(t).$

$\Rightarrow$

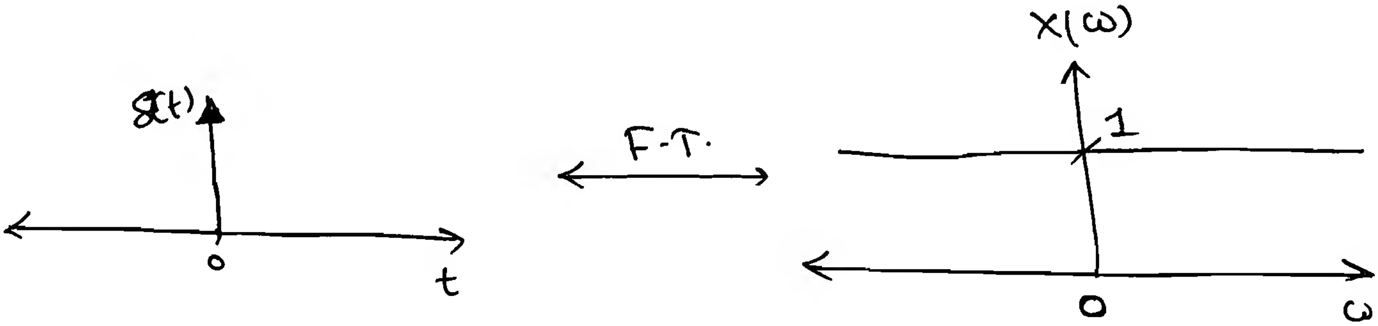
$$X(\omega) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t} \cdot dt$$

$$X(\omega) = e^{-j\omega(0)} \cdot dt. \quad (\because t_0 = 0$$

shifting property of impulse).

$$\boxed{X(\omega) = 1}$$

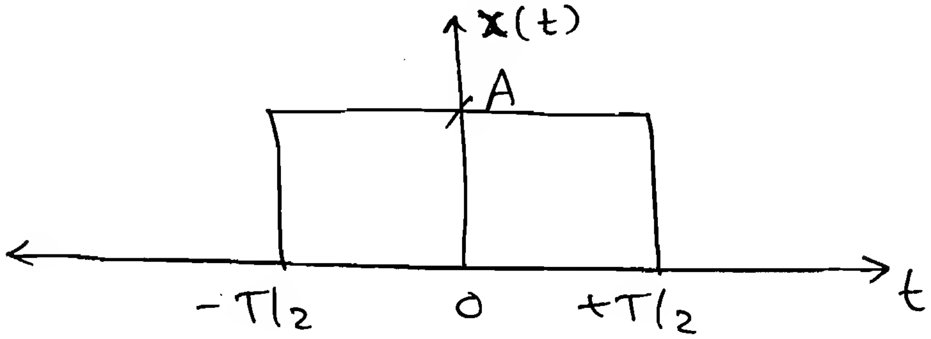
$$\therefore \quad \delta(t) \xleftrightarrow{F.T.} 1.$$



$\Rightarrow$ Spectrum of impulse is constant for all the frequency.

④ $x(t) = A\,\mathrm{rect}(t/T)$ (or) $A\,\Pi(t/T)$.

$\Rightarrow$



$$\Rightarrow \quad X(\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} \, dt.$$

$$\therefore \quad X(\omega) = \int_{-T/2}^{+T/2} A \cdot e^{-j\omega t} \, dt.$$

$$= A \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-T/2}^{+T/2}.$$

$$\therefore \quad X(\omega) = \frac{A}{j\omega} \left[e^{\frac{j\omega T}{2}} - e^{-\frac{j\omega T}{2}} \right].$$

$$= \frac{A}{j\omega} \times 2j \left[\frac{e^{j\frac{\omega T}{2}} - e^{-j\frac{\omega T}{2}}}{2j} \right].$$

$$= \frac{2A}{\omega} \times \sin\frac{\omega T}{2}.$$

$$= \frac{2A}{\omega} \times \frac{1}{\frac{\omega T}{2}} \times \frac{\omega T}{2} \quad \sin\frac{\omega T}{2}.$$

$$= \frac{2A}{\cancel{\omega}} \times \frac{\cancel{\omega} T}{\cancel{2}} \times \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}.$$

$$\therefore \boxed{X(\omega) = AT\, sa\left(\frac{\omega T}{2}\right).}$$

$\Rightarrow$

$$Sa(x) = \frac{\sin(x)}{x}$$

$$\mathrm{Sinc}(x) = \frac{\sin(\pi x)}{x}. = \frac{\sin \pi x}{\pi x}.$$

$$\Rightarrow \mathrm{Sinc}\left(\frac{x}{\pi}\right) = \frac{\sin\left(\frac{\cancel{\pi} x}{\cancel{\pi}}\right)}{\frac{\cancel{\pi} x}{\cancel{\pi}}}$$

$$= \frac{\sin x}{x} = Sa(x).$$

$$\Rightarrow \boxed{\mathrm{sinc}\left(\frac{x}{\pi}\right) = Sa(x).}$$

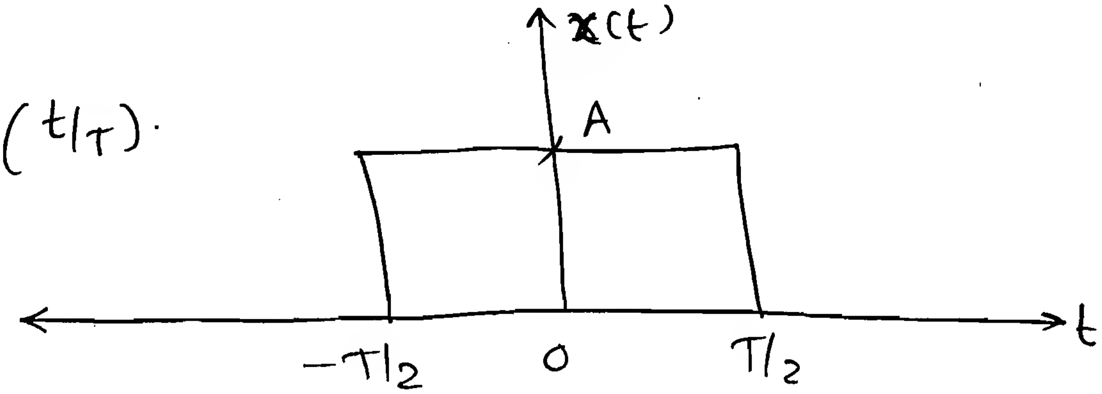
$$\therefore \boxed{X(\omega) = AT\, \mathrm{sinc}\left(\frac{\omega T}{2\pi}\right).}$$

$$= AT\, \mathrm{sinc}\,(fT). \qquad \left(\because \omega = \frac{2\pi}{t}\right).$$

$\Rightarrow$

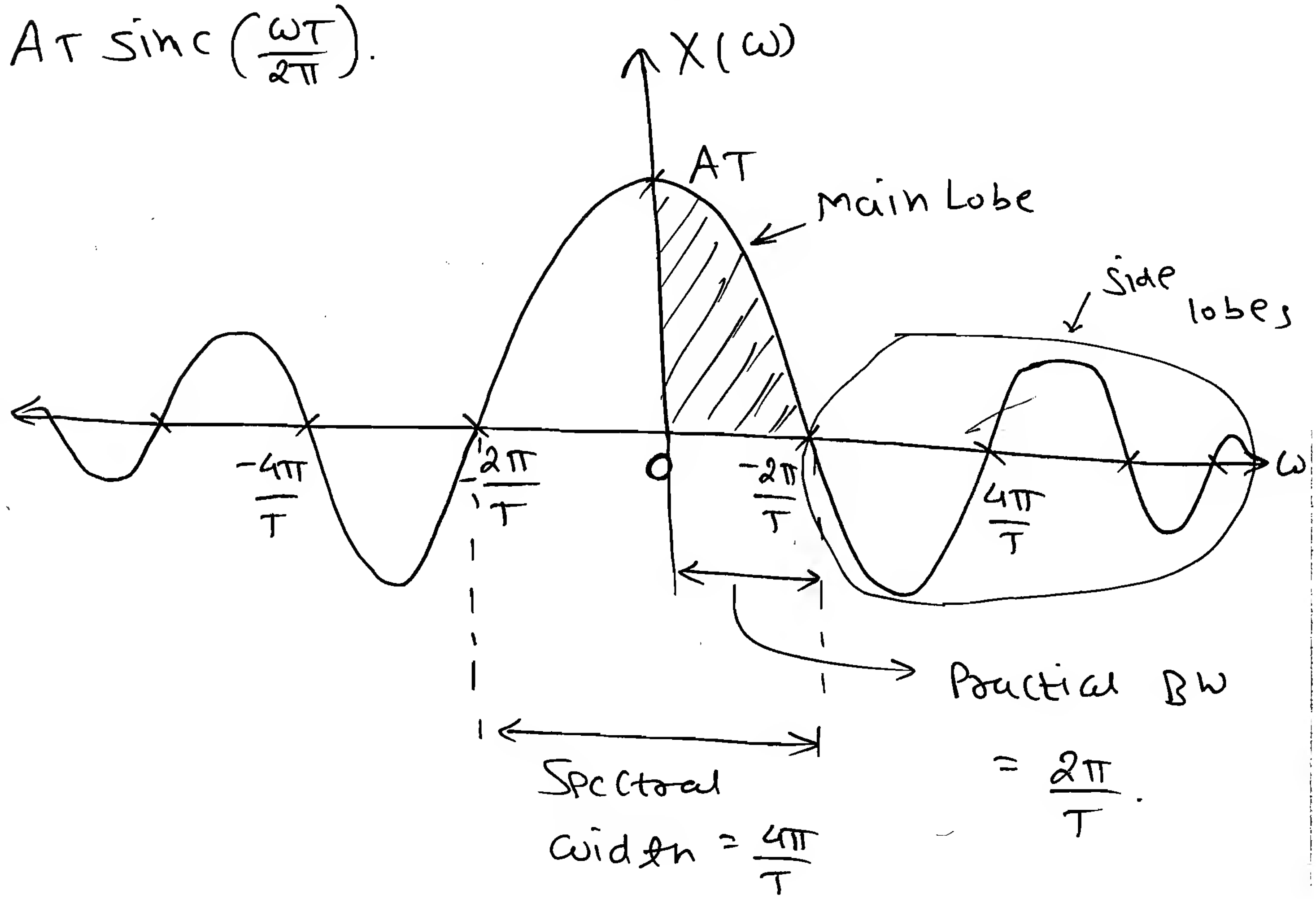
$A\,\mathrm{rect}(t/T)$.

$\Big\downarrow$ F.T.



$\Downarrow$ F.T.

$AT\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$.



$\Rightarrow$ Area under spectrum is

$\Rightarrow$ Value (Amp) of signal at $t = 0$.

$\Rightarrow$ Area under signal $\Rightarrow$ Value (Amp) of spectrum at ~~$f=0$~~ $\omega = 0$.

⇒ Practical $BW = \frac{2\pi}{T}$.

Spectral width $= \frac{4\pi}{T}$.

⇒ Signal is zero at multiple of $\frac{2\pi}{T}$ of ω.

⇒ $\boxed{\text{Null to Null BW} = \text{Zero crossing BW} \Rightarrow \frac{2\pi}{T}}$

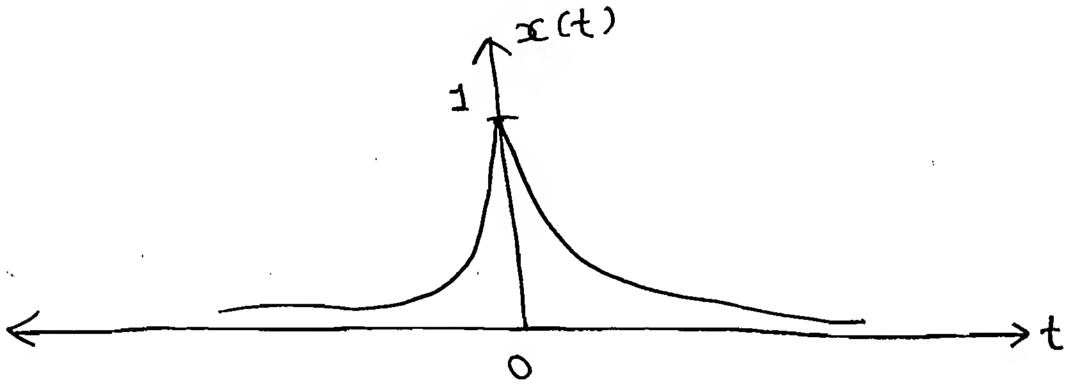
⇒ A signal can not be time limited and Band limited simultaneously.

⇒ Narrow Band in one domain becomes wideband in other domain.

(5) $x(t) = e^{-\alpha|t|}$

⇒ $x(t) = e^{-\alpha t}$; $t > 0$

$= e^{\alpha t}$; $t < 0$.



$$\therefore \quad y(t) = e^{-\alpha t} \cdot u(t) + e^{\alpha t} \cdot u(-t).$$

$$= \frac{1}{\alpha + j\omega} + \frac{1}{\alpha - j\omega}.$$

$$y(t) = \frac{2\alpha}{\alpha^2 + \omega^2}.$$

$$\therefore \quad \boxed{e^{-\alpha|t|} \xleftrightarrow{F.T.} \frac{2\alpha}{\alpha^2 + \omega^2}.}$$

Method - II:

$\Rightarrow$ real $\longleftrightarrow$ even.

e.g. $x(t) = e^{-\alpha t} \cdot u(t).$

$$X(\omega) = \frac{1}{\alpha + j\omega} \times \frac{\alpha - j\omega}{\alpha - j\omega}.$$

$$X(\omega) = \frac{\alpha - j\omega}{\alpha^2 + \omega^2}.$$

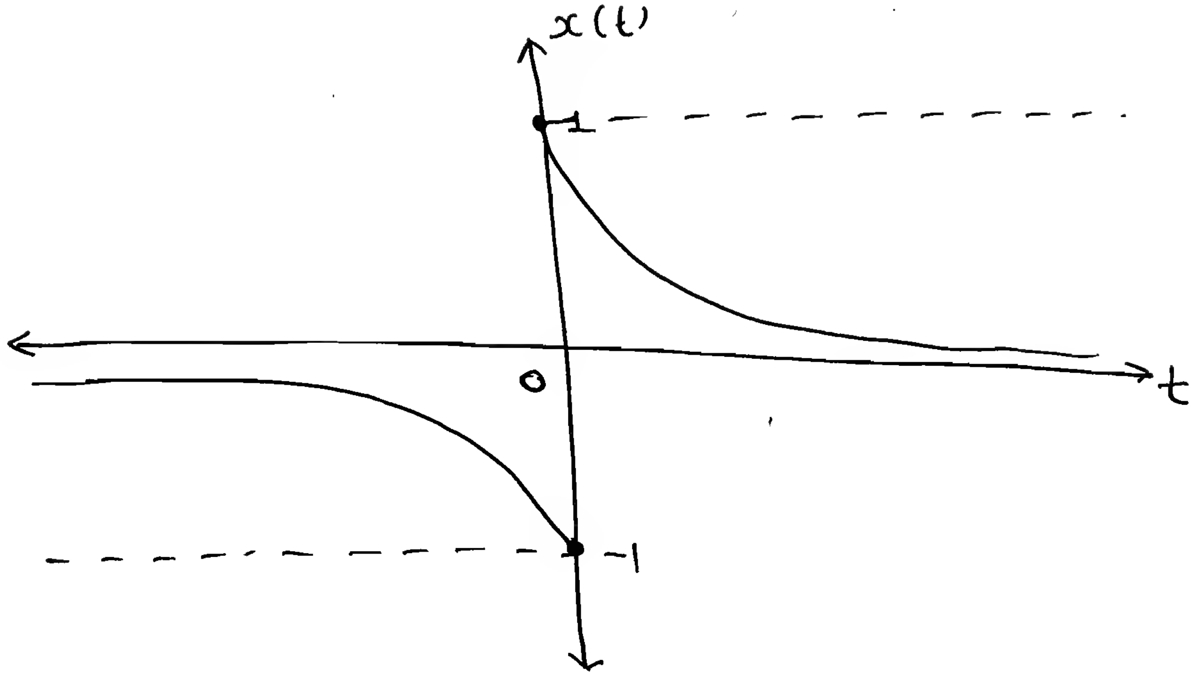
$$\Rightarrow \quad x_{even}(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{X(\omega) + X(-\omega)}{2}$$

$$\boxed{X_{even}(t). = \frac{\alpha}{\alpha^2 + \omega^2}}$$

⑥ $x(t) = e^{-\alpha t} u(t) - e^{\alpha t} \cdot u(-t)$.

$\Rightarrow$



$$\Rightarrow \quad X(\omega) = \frac{\alpha - j\omega}{\alpha^2+\omega^2} - \frac{\alpha + j\omega}{\alpha^2+\omega^2}.$$

$$\boxed{X(\omega) = \frac{-2j\omega}{\alpha^2+\omega^2}} \Rightarrow \text{real} \longleftrightarrow \text{odd}.$$

Now, say $\alpha \to 0$

$$\lim_{\alpha\to 0} X(\omega) = \lim_{\alpha\to 0} \frac{-2j\omega}{\alpha^2+\omega^2} = \frac{-2j\cancel{\omega}}{\omega^{\cancel{2}}}$$

$$= \frac{2}{j\omega}.$$

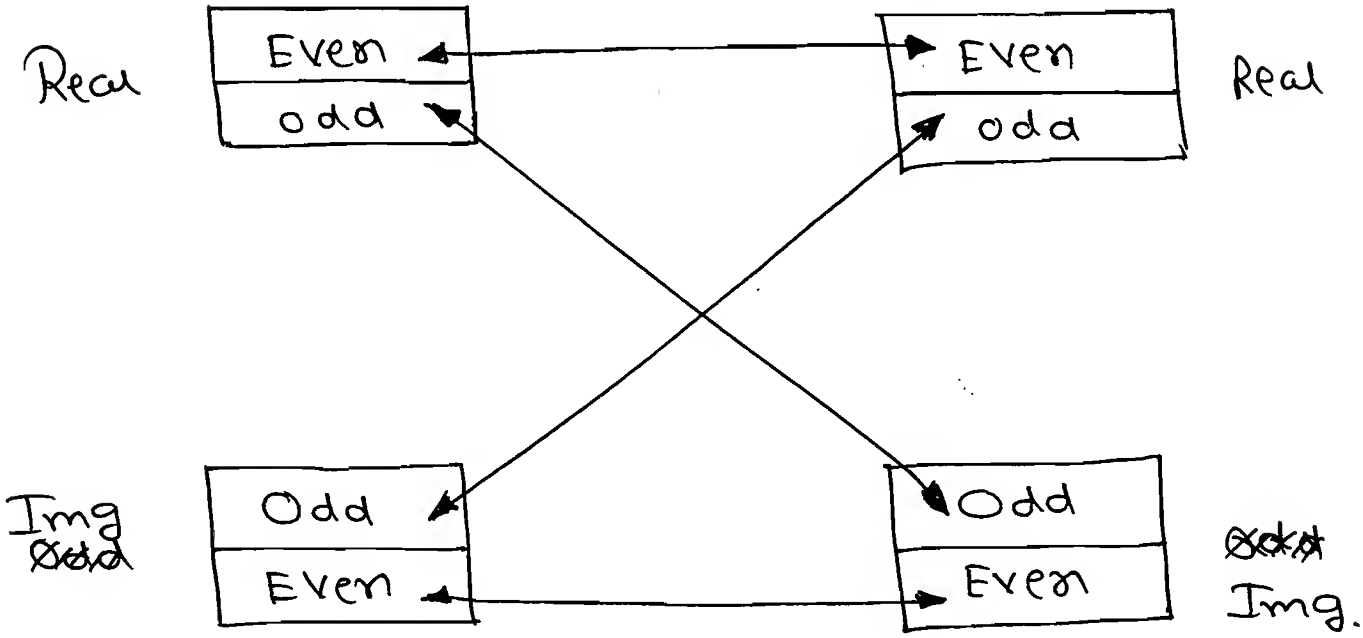
Now, $\lim_{\alpha \to 0} y(t) = \lim_{\alpha\to 0} e^{-\alpha t} \cdot u(t) - e^{\alpha t} \cdot u(-t)$.

$= u(t) - u(-t). = \mathrm{sgn}(t)$.

$\Rightarrow$ $$\boxed{\mathrm{Sgn}(t) \xleftrightarrow{\text{F.T.}} \frac{2}{j\omega}}$$

real & odd $\qquad\qquad$ Imag & odd.

*



* Properties of F.T. :

① Duality:

$\Rightarrow$ $$x(t) \longleftrightarrow X(\omega).$$

$$X(t) \longleftrightarrow 2\pi\, x(-\omega).$$

$\Rightarrow$ $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} \cdot d\omega.$$

replace 't' by '-t'.

$$\therefore \quad x(-t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{-j\omega t} \cdot d\omega.$$

$$\therefore \quad 2\pi x(-t) = \int_{-\infty}^{+\infty} X(\omega) \cdot e^{-j\omega t} \cdot d\omega.$$

$t \leftrightarrow \omega$

$$\therefore \quad 2\pi x(-\omega) = \int_{-\infty}^{\infty} X(t) \cdot e^{-j\omega t} \cdot dt.$$

$$\boxed{2\pi x(-\omega) = X(t).}$$

$\boxed{\cancel{\text{Q4}}}$ e.g.:

i) $$e^{3t} u(-t) \longleftrightarrow \frac{1}{3-j\omega}.$$

$$\frac{1}{3-jt} \xleftrightarrow{t=-\omega} 2\pi e^{3(-\omega)} \cdot u(\omega).$$

ii) $$e^{-|t|} \longleftrightarrow \frac{2}{\omega^2+1}.$$

$$\frac{2}{t^2+1} \longleftrightarrow 2\pi\, e^{-|-\omega|}$$

$$\Rightarrow 2\pi e^{-|\omega|} \;//$$

iii) $$\delta(t) \longleftrightarrow 1$$

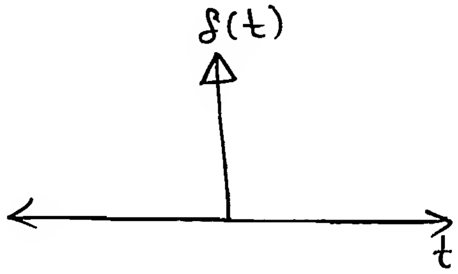
$$1 \longleftrightarrow 2\pi\, \delta(-\omega)$$

$$= \frac{2\pi}{|-1|} \cdot \delta(\omega)$$

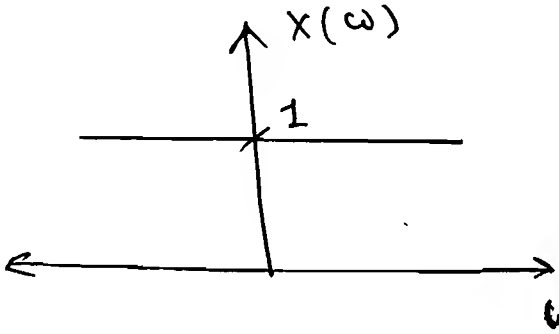
$$= 2\pi \cdot \delta(\omega).$$

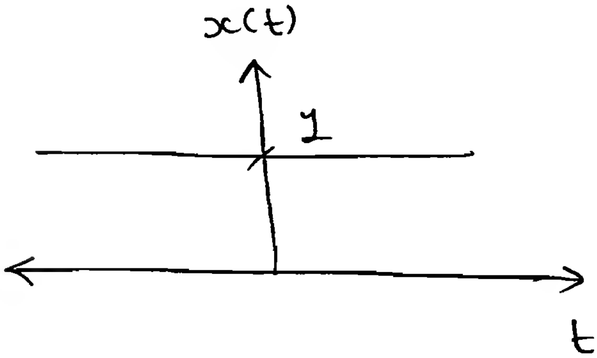
$$= \delta(f).$$

⇒

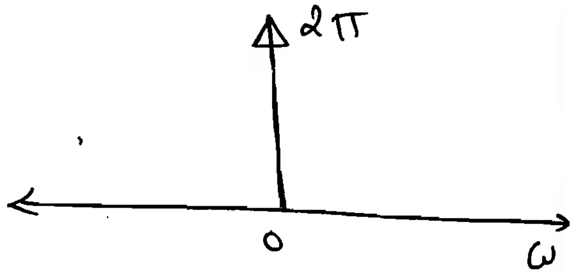


$\xleftrightarrow{F.T.}$





$\xleftrightarrow{F.T.}$



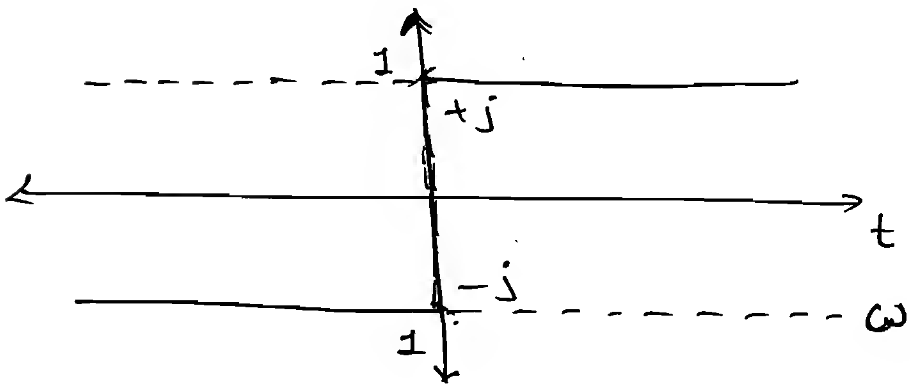
(iv)

$$\text{Sgn}(t) \xleftrightarrow{F.T} \frac{2}{j\omega}.$$

$$\frac{\cancel{2}}{jt} \xleftrightarrow{F.T} \cancel{2}\pi\ \text{Sgn}(-\omega)$$

$$\frac{1}{jt} \longleftrightarrow -\pi\ \text{Sgn}(\omega).$$

$$\therefore \boxed{\frac{1}{\pi t} \xleftrightarrow{F.T.} -j\ \text{sgn}(\omega).}$$



⇒ Impulse response of Hilbert Transform.

⇒ To get the additional $\pi/2$ phase angle we have to design $\frac{1}{\pi t}$ system in time domain.

$\Rightarrow \quad \mathrm{sgn}(t) = 2u(t) - 1.$

$\therefore \quad u(t) = \dfrac{1 + \mathrm{sgn}(t)}{2}.$

$$\therefore \quad u(t) \xleftrightarrow{F.T.} \frac{2\pi\,\delta(\omega) + \frac{2}{j\omega}}{2}$$

$$\Rightarrow \quad \boxed{u(t) \xleftrightarrow{F.T.} \frac{1}{j\omega} + \pi\,\delta(\omega).}$$

$\Rightarrow$ rect in time domain $\longleftrightarrow$ Sa in freq.

$$\mathrm{rect}\left(\frac{t}{2a}\right) \longleftrightarrow 1.\,2a\ \mathrm{Sa}\left(\frac{\omega 2a}{2}\right).$$

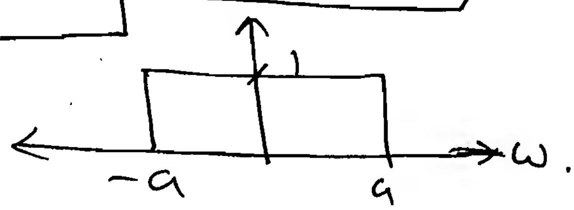
$$\mathrm{rect}\left(\frac{t}{2a}\right) \longleftrightarrow 2a.\ \mathrm{Sa}(\omega a).$$

Duality

$$2a.\ \mathrm{Sa}(at) \longleftrightarrow 2\pi.\mathrm{rect}\left(-\frac{\omega}{2a}\right).$$

$$a.\ \frac{\sin(at)}{at} \longleftrightarrow \pi.\ \mathrm{rect}\left(\omega/2a\right). \quad \text{(even)}.$$

$$\therefore \quad \boxed{\frac{\sin(at)}{\pi t} \longleftrightarrow \mathrm{rect}\left(\omega/2a\right).}$$



② Time Scaling :-

$\Rightarrow$ If $\quad x(t) \longleftrightarrow X(\omega)$.

$$\boxed{x(\alpha t) \longleftrightarrow \frac{1}{|\alpha|} \cdot X(\omega/\alpha).}$$

Compression $\longleftrightarrow$ Expansion.

e.g. let, $\quad x(t) = A \, rect \, (t/T)$.

$\therefore \quad A \, rect \, (t/T) \longleftrightarrow AT \, Sa \left(\frac{\omega T}{2}\right)$.

Now, $\quad y(t) = A \, rect \left(\frac{2t}{T}\right)$.

$y(t) = x(2t)$.

$$\therefore \quad Y(\omega) = \frac{1}{2} X(\omega/2).$$

$$= \frac{AT}{2} \, Sa \left(\frac{\frac{\omega}{2} \cdot T}{2}\right).$$

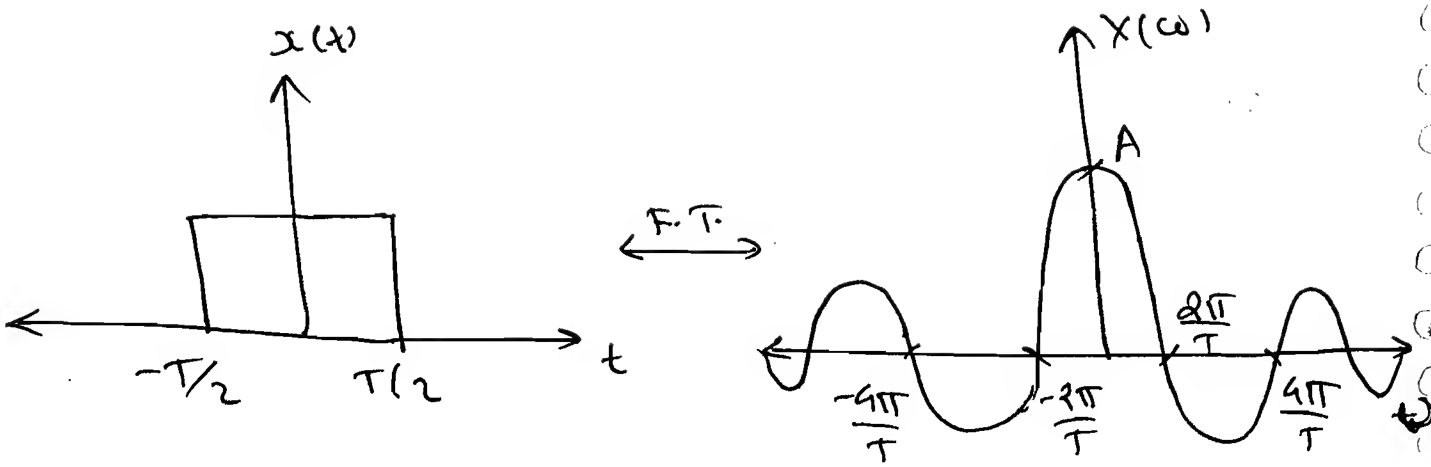
$$Y(\omega) = \frac{AT}{2} \, Sa \left(\frac{\omega T}{4}\right).$$

$$\therefore \quad \frac{\omega T}{4} = \pm n\pi \Rightarrow \omega = \pm \frac{4n\pi}{T}.$$

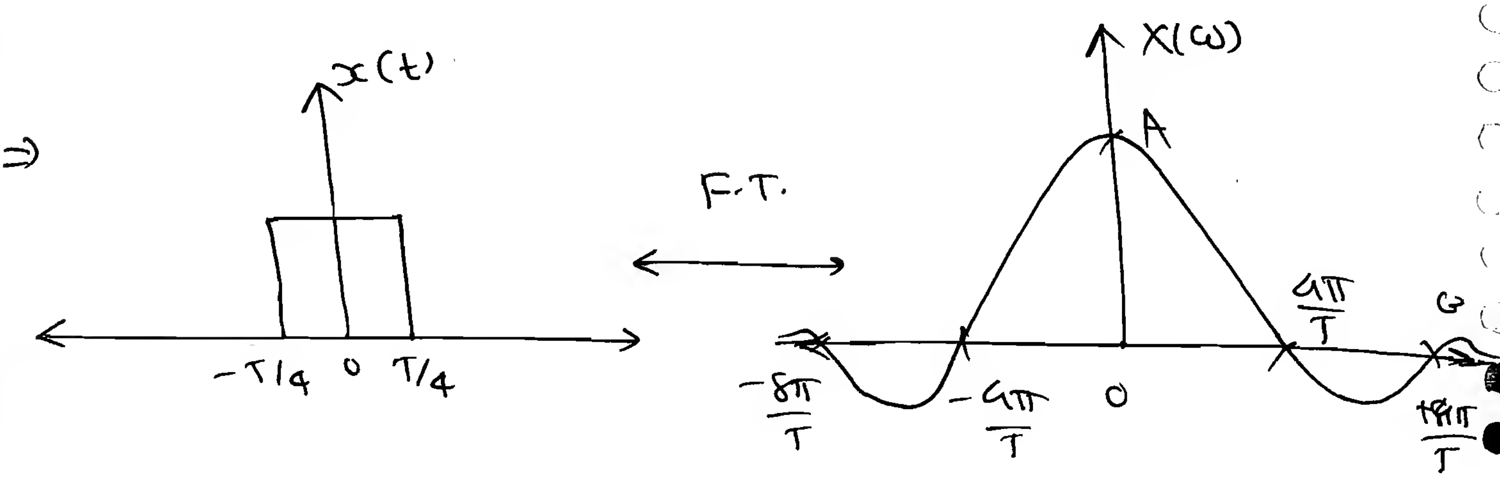
$$\therefore \quad Y(\omega) = \frac{AT}{2} \, sinc \left(\frac{\omega T}{4\pi}\right).$$

$$\omega = \pm \frac{4n\pi}{T}.$$

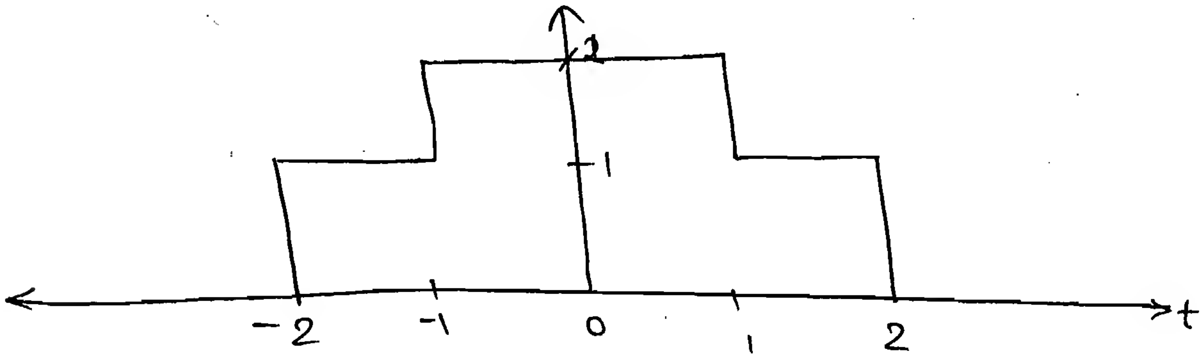
⇒



⇒

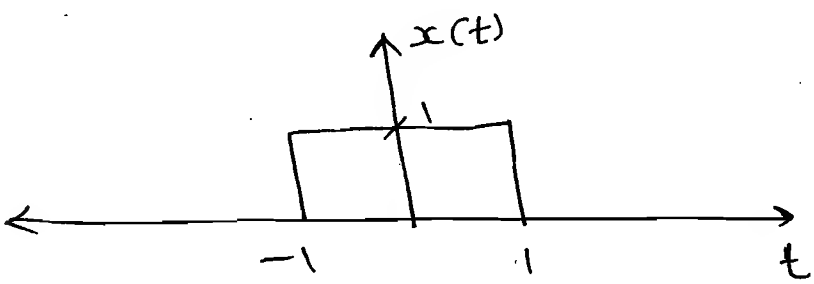


P4.2.4. Find the F.T. of the signal shown in fig?



Soln:

let, $x(t) = 1.\,\mathrm{rect}(t/2)$.



$\therefore\ y(t) = \cancel{2}\, x(t) + x(t/2)$.

$\therefore \quad 1.\ \mathrm{rect}\,(t/2) \longleftrightarrow 2\,Sa\left(\frac{2\omega}{2}\right)$

$\Rightarrow 2\,Sa(\omega).$

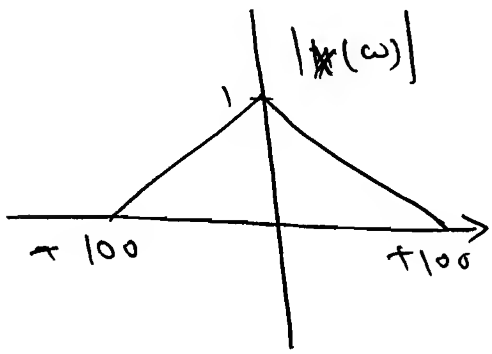
$\therefore \quad y(t) = \cancel{2}\,x(t) + x(t/2).$

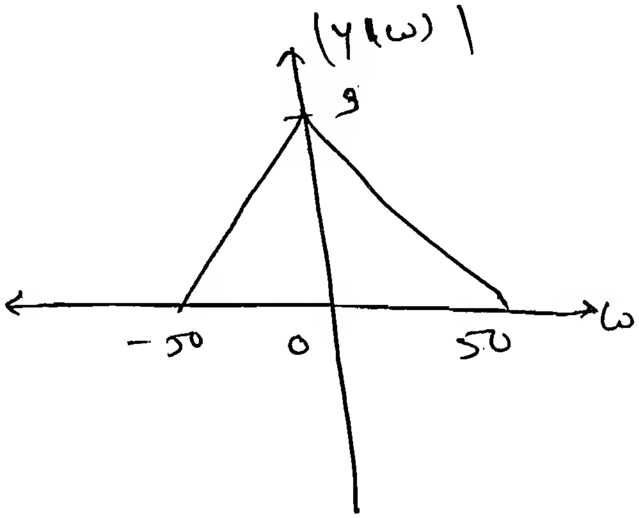
$\therefore \quad y(t) = \overset{2}{\cancel{4}}\,Sa(\omega) + \frac{1}{|1/2|} \cdot X(2\omega/\cancel{2}).$

$\therefore \quad Y(\omega) = \overset{2}{\cancel{4}}\,Sa(\omega) + 2\cancel{2}\,Sa(2\omega/\cancel{2}).$

$\therefore \quad \boxed{Y(\omega) = 2Sa(\omega) + 4\,Sa(2\omega).}$

$\boxed{P4.2.5}$ The magnitude of F.T. $X(\omega)$ of a function in fig (a) the magnitude of F.T. $Y(\omega)$ of other function $y(t)$ is shown below in fig (b). The phases $X(\omega)$ and $Y(\omega)$ are zero for all ω. The magnitude and frequency units are identical in both the figures. The f^n $y(t)$ can be expressed in terms of $x(t)$ a) ________





Soln:

$$|Y(\omega)| = 3|X(2\omega)|$$

$$= 3 \cdot X(\omega/\tfrac{1}{2})$$

$$= 3 \cdot \tfrac{1}{2} \times \frac{1}{1/2} X(\omega/\tfrac{1}{2})$$

$$\therefore \boxed{y(t) = \frac{3}{2} x(t/2)}$$

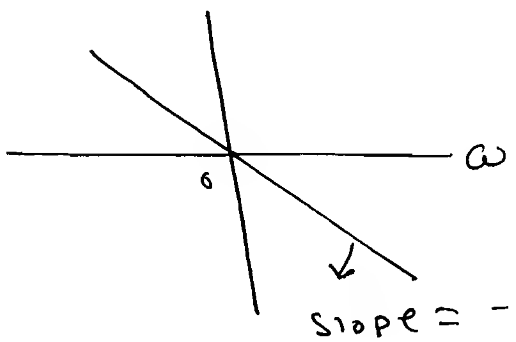
③ Time - Shifting:

$\Rightarrow$ If $x(t) \xleftrightarrow{F.T.} X(\omega)$

then $\boxed{x(t-t_0) \xleftrightarrow{F.T.} e^{-j\omega t_0} X(\omega)}$

$\Rightarrow$ Time-delay in a signal causes a linear phase shift in its spectrum. Shifting in time doesn't alter the amplitude spectrum of signal.

$$\therefore \theta(\omega) = -\omega t_0$$



e.g. ① $y(t) = e^{2t} \cdot \cancel{x(t)} \cdot \cancel{u} \; u(-t+3)$.

Soln: $y(t) = e^{2(t-3)+6} \cdot u\,[-(t-3)]$.

$$\therefore \; Y(\omega) = e^{6}\left[\frac{e^{-j\omega(3)}}{2-j\omega}\right].$$

(2) $y(t) = rect\left(\frac{t+1}{4}\right)$

Soln:

let, $x(t) = rect\,(t/4)$. , $A=1$, $T=4$.

$\therefore \; y(t) = x(t+1)$. where,

$t_0 = -1$.

↓ F.T.

$$Y(\omega) = e^{j\omega}\cdot X(\omega).$$

& $X(\omega) = 4.\,Sa\left(\frac{4\omega}{2}\right)$.

$$\therefore \; \boxed{Y(\omega) = 4e^{j\omega}\cdot Sa(2\omega).}$$

✓ *

[a] given $H(\omega) = \frac{4\sin 2\omega.\cos\omega}{\omega}$, find $h(0)$.

Soln:

$$H(\omega) = \frac{4\sin 2\omega.\cos\omega}{\omega}$$

$$= 2\left[\frac{2\sin 2\omega}{\omega}\right].\cos\omega \times 2$$

$$= 2\left[\frac{2\sin\omega.\cos\omega.\cos\omega}{\omega}\right].\times 2$$

$$= 2\left[\frac{2\sin\omega}{\omega}\right]\cos^2\omega.\times 2$$

$$= 2\left[\frac{2\sin\omega}{\omega}\right].\left[\frac{1+\cos 2\omega}{2}\right].\times 2$$

$$= \frac{2\sin\omega}{\omega}\times 2 + 2\left[\frac{2\sin\omega}{\omega}\right]\times\left[\frac{e^{-j2\omega}+e^{j2\omega}}{2}\right].$$

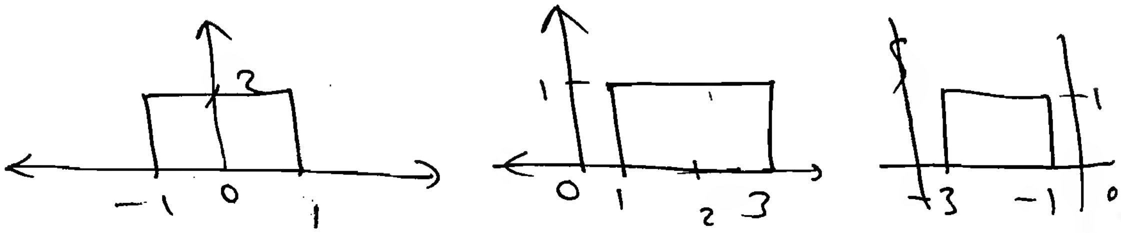
let, $x(t) = \text{rect}(t/2)$

$$\Rightarrow X(\omega) = \frac{2\sin\omega}{\omega}.$$

$$\therefore H(\omega) = 2X(\omega) + \cancel{\tfrac{1}{2}X(\omega)}\, e^{-j\omega 2}\cdot X(\omega) + e^{j2\omega}\cdot X(\omega).$$

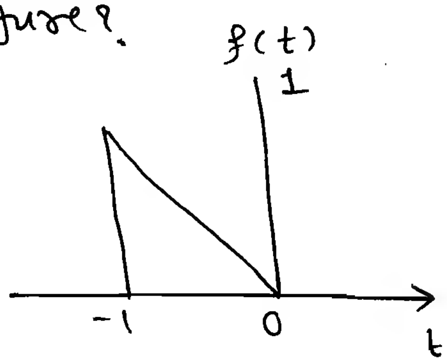
$\downarrow$ IFT

$$\therefore h(t) = 2x(t) + x(t-2) + x(t+2).$$

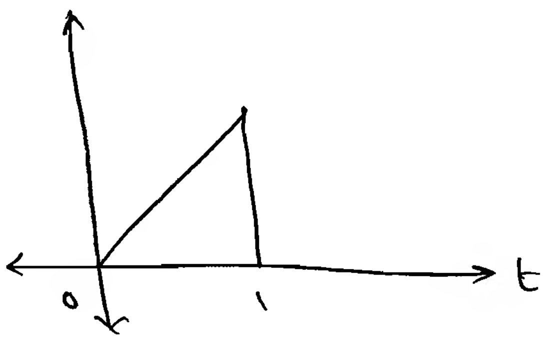


$$\therefore \boxed{h(0) = 2}$$

$\boxed{P4.2.11}$: The F.T. of a triangular pulse $f(t)$ shown in figure $F(\omega) = \dfrac{e^{j\omega} - j\omega e^{j\omega} - 1}{\omega^2}$ using this find the F.T. of the signals shown in figure?



Solⁿ: ① $f_1(t)$

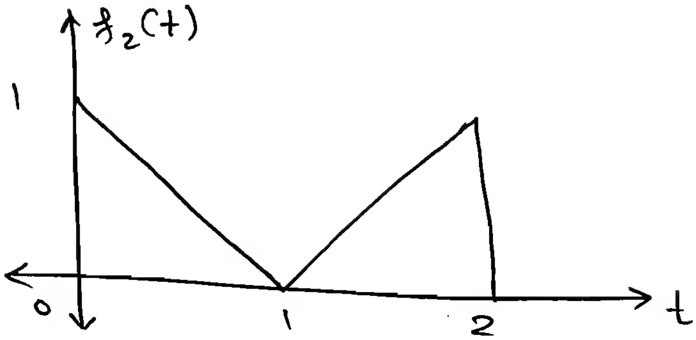


$$\Rightarrow \quad f_1(t) = f(-t).$$

$$\downarrow$$

$$F_1(\omega) = F(-\omega) = \frac{e^{-j\omega} + j\omega e^{-j\omega} - 1}{\omega^2}.$$

(2)



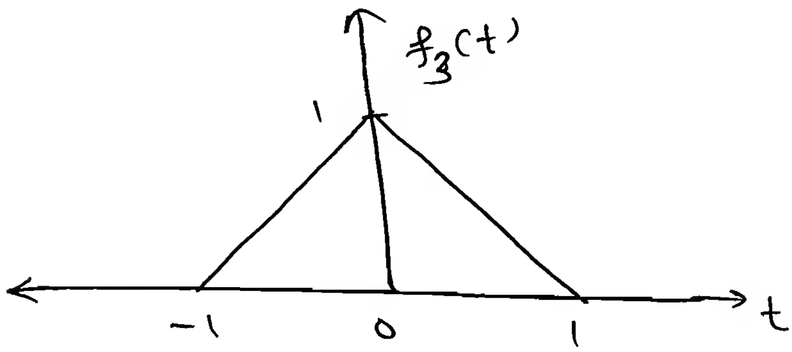
Soln:

$$f_2(t) = f_1(t-1) + f(t-1).$$

$$\downarrow \text{F.T.}$$

$$\therefore \quad F_2(\omega) = e^{-j\omega} F_1(\omega) + e^{-j\omega} F_2(\omega).$$

(3)

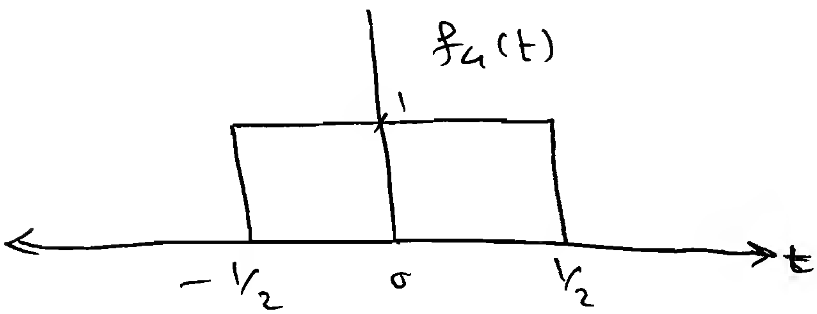


Soln:

$$f_3(t) = f_1(t+1) + f(t-1).$$

$$F_3(\omega) = e^{j\omega} F_1(\omega) + e^{-j\omega} F(\omega).$$

(4)

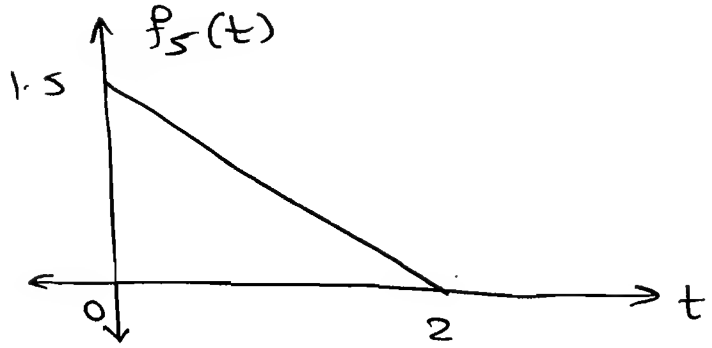


Sol^n:

$$f_4(t) = f(t - \tfrac{1}{2}) + f_1(t + \tfrac{1}{2}).$$

$$\therefore F_4(\omega) = e^{-j\frac{1}{2}\omega} \cdot F(\omega) + e^{+j\frac{\omega}{2}} \cdot F_1(\omega).$$

(5)



Sol^n:

$$f_5(t) = 1.5 f\left(\tfrac{t}{2} - 1\right).$$

$$= 1.5 f\left(\frac{t-2}{2}\right). \quad t_0 = 2.$$

$$\alpha = \tfrac{1}{2}$$

$$\therefore F_5(\omega) = \frac{1.5}{\frac{1}{2}} \cdot e^{-j2\omega} \cdot F(\omega/\tfrac{1}{2}).$$

$$\therefore \boxed{F_5(\omega) = 3 \cdot e^{-j2\omega} \cdot F(2\omega).}$$

(4) Frequency Shifting:

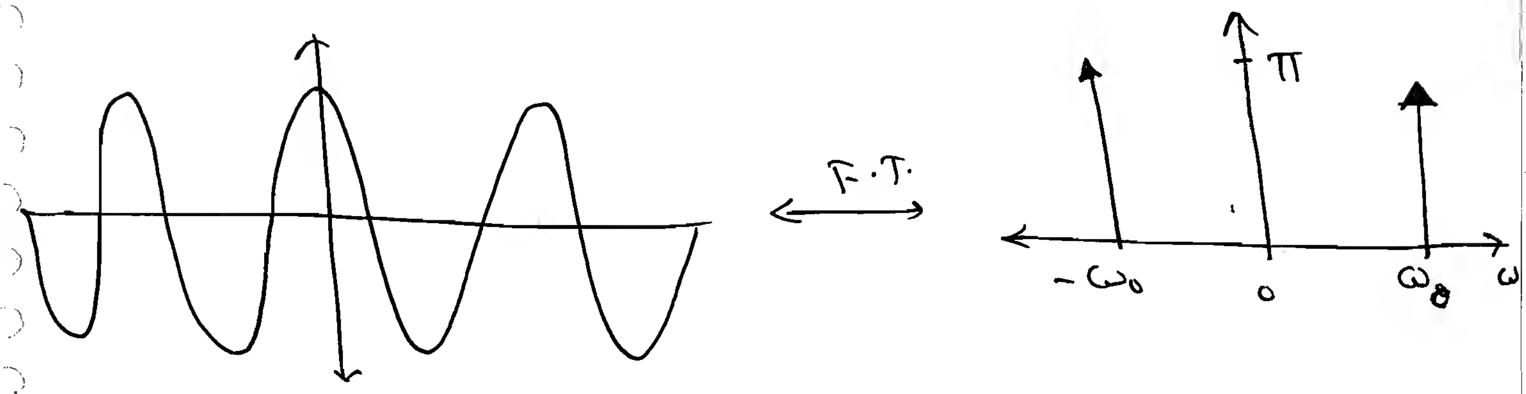
$\Rightarrow$ If $x(t) \longleftrightarrow X(\omega)$.

then $x(t) \cdot e^{j\omega_c t} \xleftrightarrow{F.T.} X(\omega - \omega_c)$.

e.g. $\cos\omega_0 t = \dfrac{1 \cdot e^{+j\omega_0 t} + 1 \cdot e^{-j\omega_0 t}}{2}$.

$$\xleftrightarrow{F.T} = \frac{2\pi\delta(\omega \mp \omega_0) + 2\pi\,\delta(\omega + \omega_0)}{2}.$$

∴ $\cos\omega_0 t \xleftrightarrow{F.T.} \dfrac{\delta(f-f_c)+\delta(f+f_c)}{2}$.



⇒ $e^{-3t}\cdot \sin 6t \cdot u(t)$ ⇒ damped oscillation.

↓ reference signal.

⇒ $\text{rect}(t/4) \cdot \cos 6t$ ← reference pulse.

$\boxed{Q}$ F.T. of $y(t) = \text{sinc}(t)\cdot \cos 10\pi t$.

$\underline{\text{Soln}}$:

$$y(t) = \text{sinc}(t)\cdot\left[\frac{e^{j10\pi t}+e^{-j10\pi t}}{2}\right].$$

↓F.T. $\quad$ ↓ $x(t)$

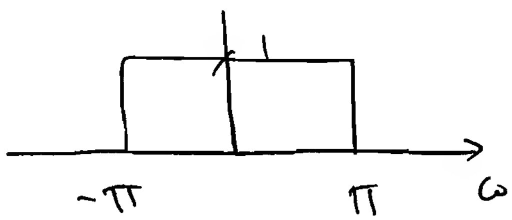
$$Y(\omega) = \frac{X(\omega-10\pi)+X(\omega+10\pi)}{2}.$$

$x(t) = \text{sinc}(t). = \dfrac{\sin \pi t}{\pi t}$ $\quad(a=\pi)$

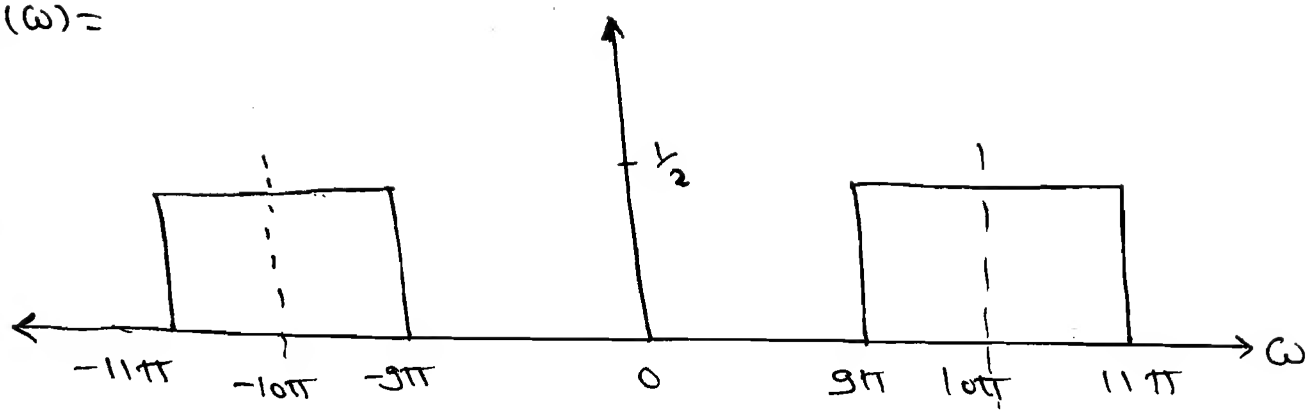
⇒ $X(\omega) = \text{rect}(\omega/2a)$.

$X(\omega) = \text{rect}(\omega/2\pi)$.

∴ $X(\omega)$ ⇒



$\therefore Y(\omega) =$



④ I.F.T. of $X(4\omega+3)$.

Soln:

$$Y(\omega) = X(4(\omega + 3/4)).$$

$$= \frac{1}{4} \cdot \frac{1}{|1/4|} \cdot X\left(\frac{\omega}{1/4}\right).$$

$$\boxed{y(t) = \frac{1}{4} \cdot x(t/4) \cdot e^{-j(3/4)t}}$$

Note:

⇒ When we perform time differentiation $\delta(\omega)$ component is loss since $j\omega\,\delta(\omega) = 0$ and the original spectrum magnitudes are increased by the factor $|\omega|$ i.e. high freq. components are more amplified.

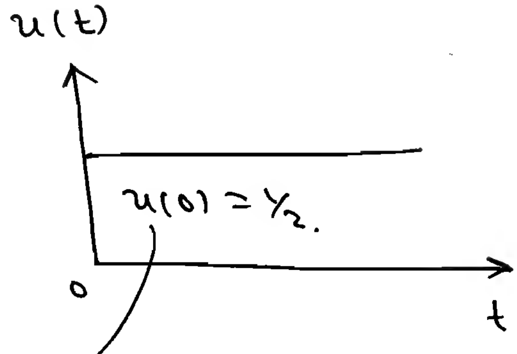
⑤ Differentiation in time:

⇒ If $x(t) \xleftrightarrow{F.T.} X(\omega)$

then $\boxed{\frac{d}{dt} x(t) \xleftrightarrow{F.T.} (j\omega) X(\omega).}$

but, $X(\omega) \neq \dfrac{F.T\left\{\frac{d}{dt}x(t)\right\}}{j\omega}$.

e.g. ① $x(t) = u(t)$.



$\frac{d}{dt}x(t) = \delta(t)$.

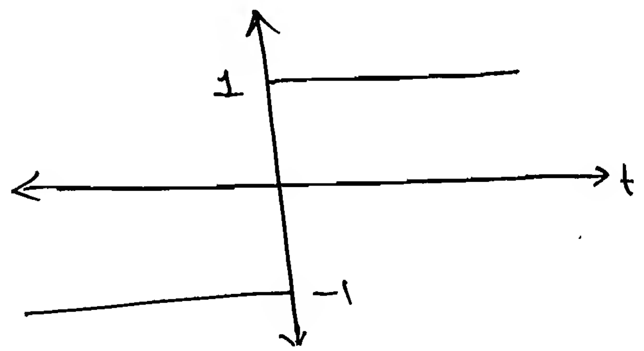
$\therefore \quad j\omega X(\omega) = 1$.

$X(\omega) = \dfrac{1}{j\omega} + \pi\delta(\omega)$.

(missed by differentiation).

$\Rightarrow \quad j\omega\,\delta(\omega) = j(0).\,\delta(\omega) = 0$.

② $x(t) = \text{Sgn}(t)$



$x(t) = 2u(t) - 1$

$\therefore \quad \frac{d}{dt}x(t) = 2\delta(t)$

$\therefore \quad j\omega X(\omega) = 2$.

$$\boxed{X(\omega) = 2/j\omega.}$$

P. 4.2.16 Find the F.T. of the signal

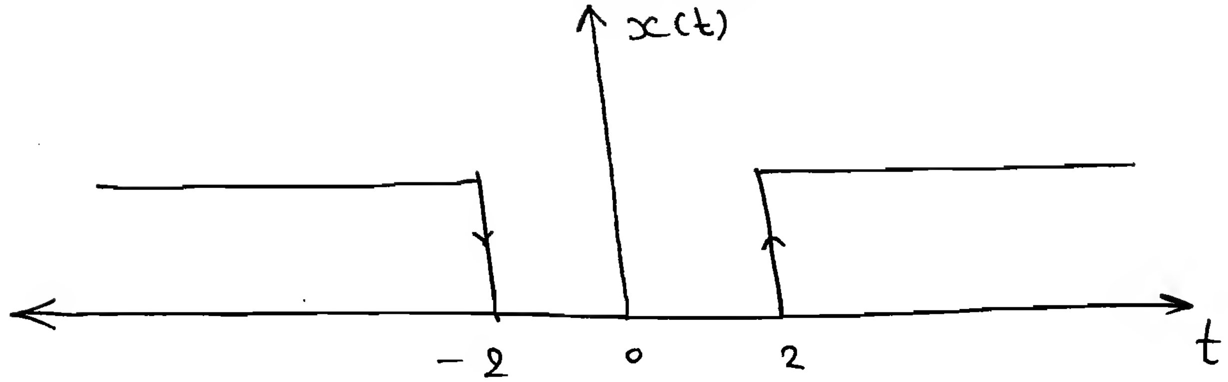
$y(t) = \frac{d}{dt}[u(t-2) + u(-t-2)]$.

Soln:

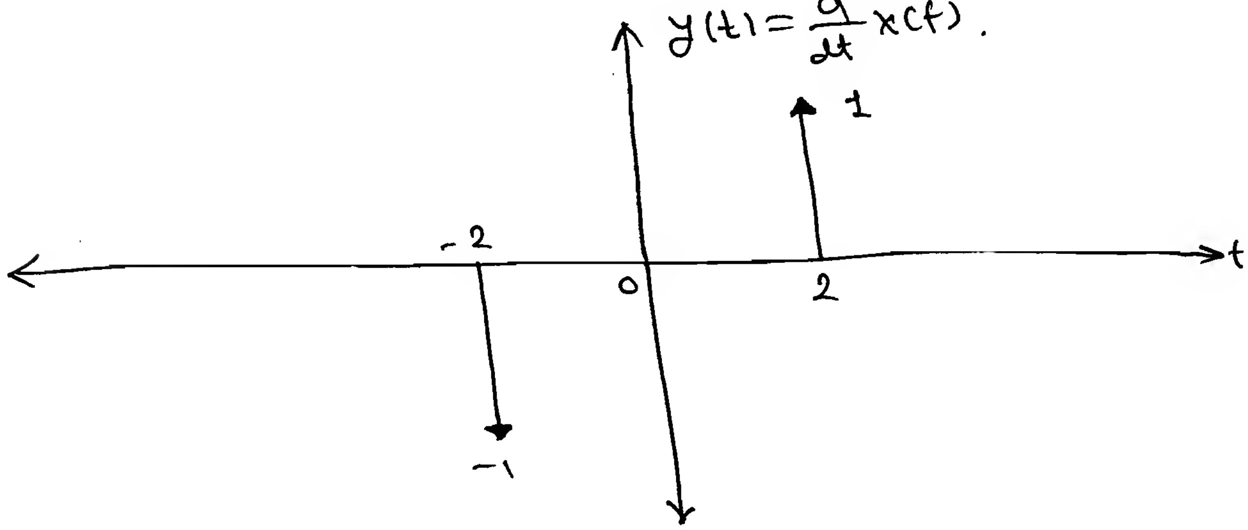
$y(t) = \frac{d}{dt}[u(t-2) + u(-t-2)]$.

$y(t) = \delta(t-2) - \delta(+t+2)$.

$\Rightarrow$



$\Downarrow$



$$\therefore \quad y(t) = -1 \cdot \delta(t+2) + \delta(t-2).$$

$$Y(\omega) = -e^{j2\omega} \cdot (1) + e^{-j2\omega} \cdot (1).$$

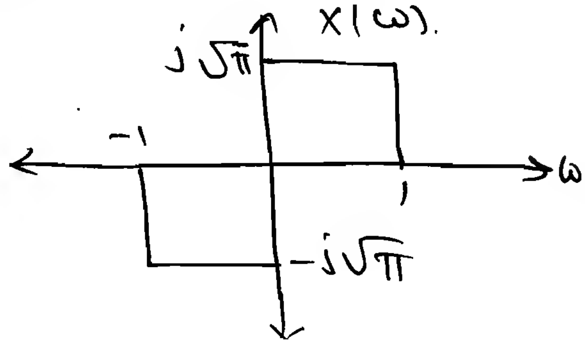
$$= -\frac{1}{2j}\left[\frac{e^{+j2\omega} - e^{-j2\omega}}{2j}\right].$$

$$= -\frac{1}{2j} \times \sin(2\omega).$$

$$\boxed{Y(\omega) = \frac{j\sin(2\omega)}{2}}$$

$\boxed{P\ 4.2.17}$ For the Speltrum $X(\omega)$ Shown in figure,

find $\frac{d}{dt}x(t)$ at $t=0$?



Soln:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} \cdot d\omega .$$

$$\therefore \frac{d x(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} (j\omega) X(\omega) \cdot e^{j\omega t} \cdot d\omega .$$

$$\left. \frac{d}{dt} x(t) \right|_{t=0} = \frac{j\omega}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot d\omega .$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} (j\omega) X(\omega) \cdot d\omega .$$

$$= \frac{1}{2\pi} \left[\int_{-1}^{0} (-j\sqrt{\pi}) (j\omega)\, d\omega + \int_{0}^{1} (j\sqrt{\pi}) (j\omega)\, d\omega \right].$$

$$= \frac{1}{2\pi} \left[\sqrt{\pi} \left(\frac{\omega^2}{2} \right)_{-1}^{0} - \sqrt{\pi} \left(\frac{\omega^2}{2} \right)_{0}^{1} \right].$$

$$= \frac{1}{2\pi} \left[-\frac{\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{2} \right].$$

$$= \frac{-\sqrt{\pi}}{2\pi}$$

$$\therefore \boxed{\left. \frac{d}{dt} x(t) \right|_{t=0} = \frac{-1}{2\sqrt{\pi}}}$$

⑥ Frequency Differentiation:

⇒ Diffn w.r.t. one variable corresponds to multiplication by other variable.

$$\boxed{-jt\,x(t) \xleftrightarrow{F.T.} \frac{d}{d\omega}X(\omega).}$$

e.g. ① $y(t) = t \cdot e^{-at} \cdot u(t).$

$$\therefore\ y(t) = \frac{-j}{-j} \cdot t \cdot e^{-at} \cdot u(t).$$

$$= \frac{-1}{j} \cdot \left(-jt \cdot e^{-at} \cdot u(t)\right).$$

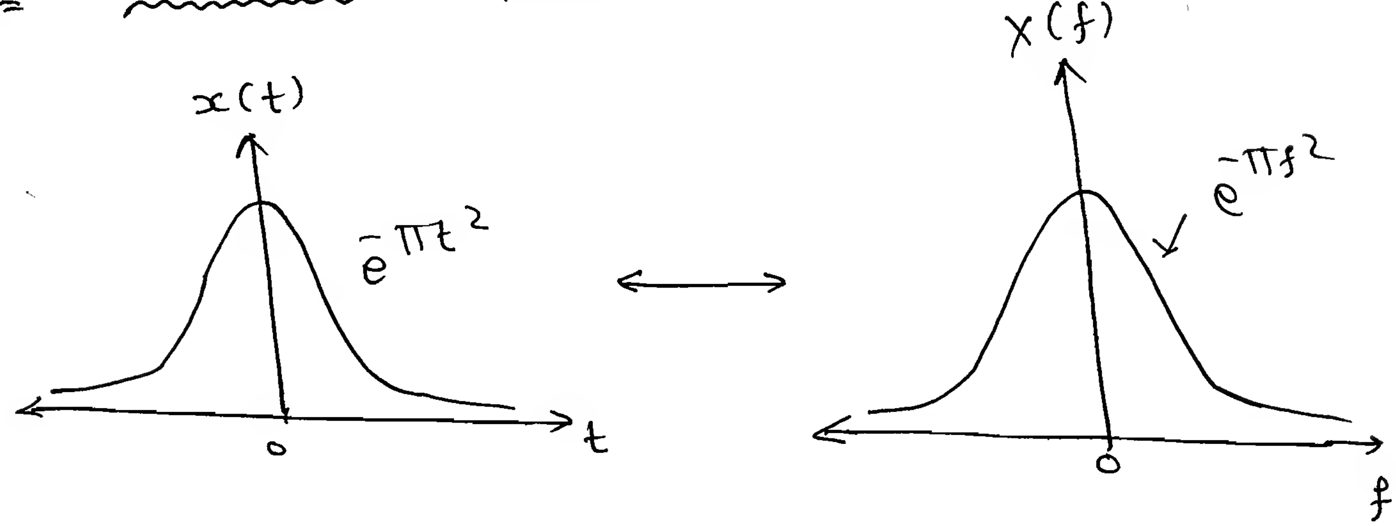
$$= +j\left[\frac{d}{d\omega} \times \frac{1}{a+j\omega}\right].$$

$$= +j \quad -\frac{1}{(a+j\omega)^2} \times (j)$$

$$\therefore\ \boxed{Y(\omega) = \frac{1}{(a+j\omega)^2}}$$

* Gaussian function:

⇒



let, $x(t) = e^{-\pi t^2}.$

$$\therefore\ \frac{d}{dt}x(t) = -e^{-\pi t^2}(2\pi t). \quad = -2\pi t\,x(t)$$

$$\therefore \boxed{\frac{d}{dt} x(t) = -2\pi t\, x(t)} \quad \text{— (I)}$$

$$\therefore \; j2\pi f\, x(t) = -2\pi t \cdot e^{-\pi t^2}$$

$$j(j2\pi f\, x(t)) = -j2\pi t \cdot e^{-\pi t^2}$$

$$\therefore \; -2\pi f \cdot x(t) = \frac{d}{df} X(f)$$

$$\therefore \boxed{\frac{d}{df} X(f) = -2\pi f\, X(f)} \quad \text{— (2)}$$

⇒ From eq I & II looking in similar manner $x(f)$ is same as $x(t)$.

⇒ In general,

$$\boxed{e^{-at^2} \; (a>0) \xleftrightarrow{F.T.} \sqrt{\frac{\pi}{a}} \cdot e^{-\omega^2/4a} \quad \text{(or)} \quad \sqrt{\frac{\pi}{a}} \cdot e^{\frac{-\pi^2 f^2}{a}}}$$

P 4.2.20 Given $x(t) \longleftrightarrow X(\omega)$, express the F.T. of the following signals in terms of $X(\omega)$?

(i) $x_1(t) = x(2-t) + x(-t-2)$.

Soln: $x_1(t) = x(-(t-2)) + x(-(t+2))$.

$$X_1(\omega) = e^{-j2\omega} \cdot X(-\omega) + e^{j2\omega} X(-\omega)$$

$$= \frac{X(-\omega)}{2} \left[\frac{e^{j2\omega} + e^{-j2\omega}}{2} \right]$$

$$\therefore \boxed{X_1(\omega) = 2\,X(-\omega)\cdot \cos 2\omega.}$$

② $x_2(t) = x(3t - 6)$.

Soln:

$$x_2(t) = x(3(t-2)).$$

$$\therefore \boxed{X_2(\omega) = \frac{1}{3}\cdot e^{-j2\omega}\cdot X(\omega/3).}$$

③ $$x_3(t) = \frac{d^2}{dt^2}x(t-3).$$

Soln:

$$x_3(t) = \frac{d^2}{dt^2}x(t-3).$$

$$X_3(\omega) = (j\omega)^2\cdot e^{-j3\omega}\cdot X(\omega).$$

$$\boxed{X_3(\omega) = -\omega^2\cdot e^{-j3\omega}\cdot X(\omega).}$$

*** ④ $$x_4(t) = t\,\frac{dx(t)}{dt}.$$

Soln:

$$x_4(t) = t\cdot\frac{dx(t)}{dt}$$

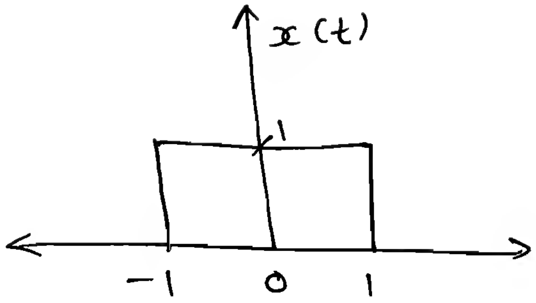
$$t \rightarrow j\frac{d}{d\omega}$$

$$X_4(\omega) = j\frac{d}{d\omega}\left(j\omega\cdot X(\omega)\right).$$

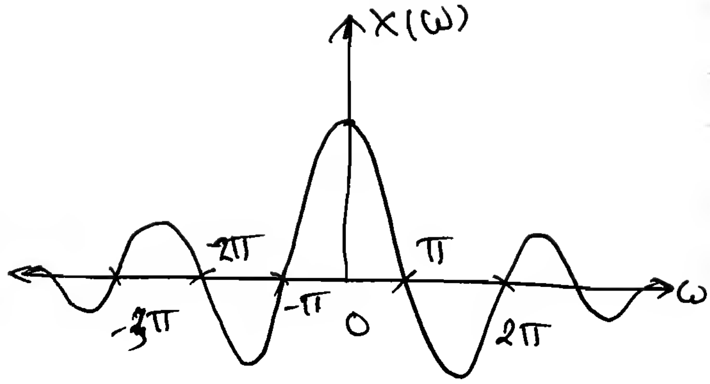
$$\therefore \boxed{X_4(\omega) = -\left(X(\omega) + \omega\frac{dX(\omega)}{d\omega}\right).}$$

$\boxed{P4.2.21}$ Given $x(t) = \begin{cases} 1; & |t|<1 \longleftrightarrow 2\frac{\sin\omega}{\omega} \\ 0; & \text{elsewhere}\end{cases}$ find the F.T. of the following signals?

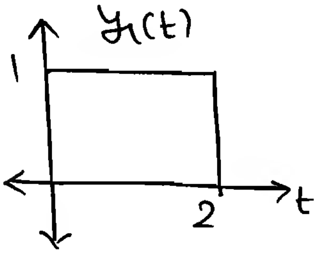
$\Rightarrow$



$\xleftrightarrow{F.T.}$



(a)

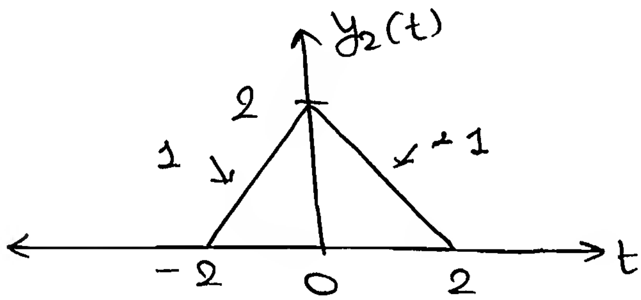


Sol^n:

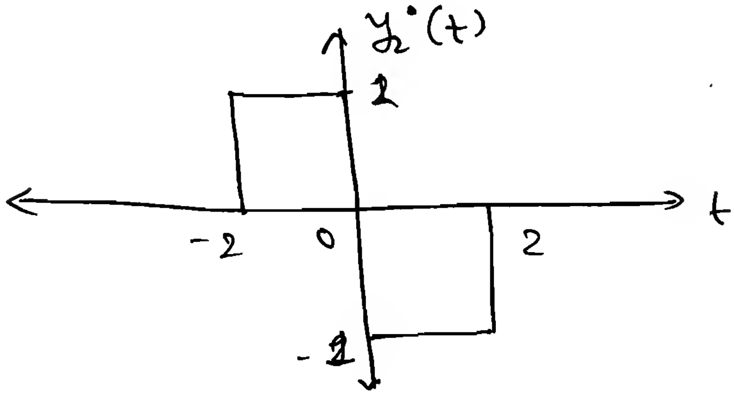
$$y_1(t) = x(t-1).$$

$$\therefore \boxed{Y_1(\omega) = e^{-j\omega} \cdot X(\omega).}$$

(b)



Sol^n.

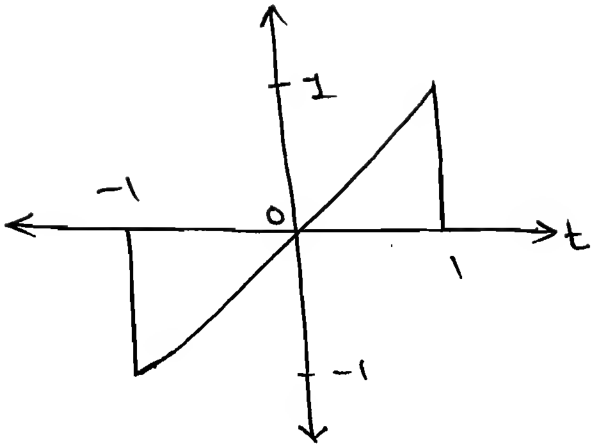


$$\therefore \quad y_2'(t) = y_1(-t) - y_1(t).$$

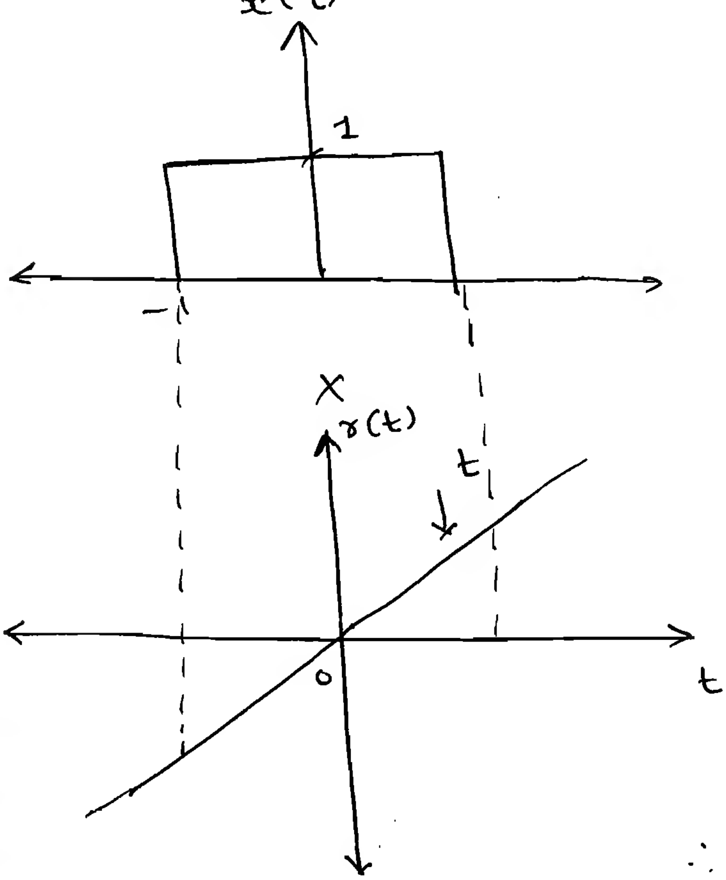
$$j\omega Y_2(\omega) = Y_1(-\omega) - Y_1(\omega).$$

$$\therefore \boxed{Y_2(\omega) = \frac{1}{j\omega}\left[Y_1(-\omega) - Y_1(\omega)\right].}$$

(c)

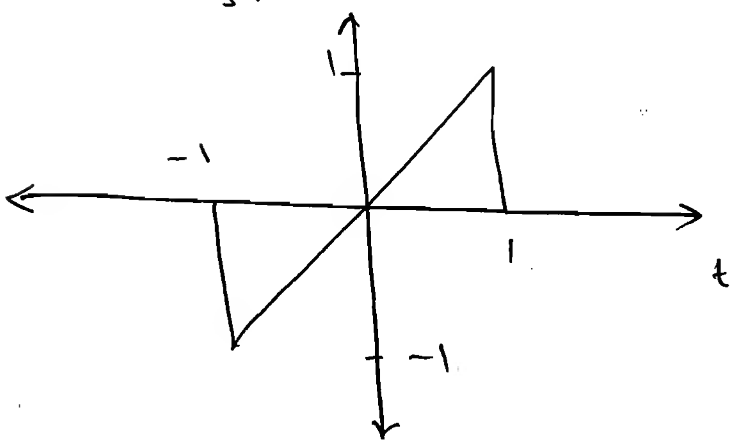


Solⁿ: Method-I



=

$y_3(t)$



So, $y_3(t) = t \cdot x(t)$.

$\therefore \quad Y_3(\omega) = j \cdot \dfrac{dX(\omega)}{d\omega}$.

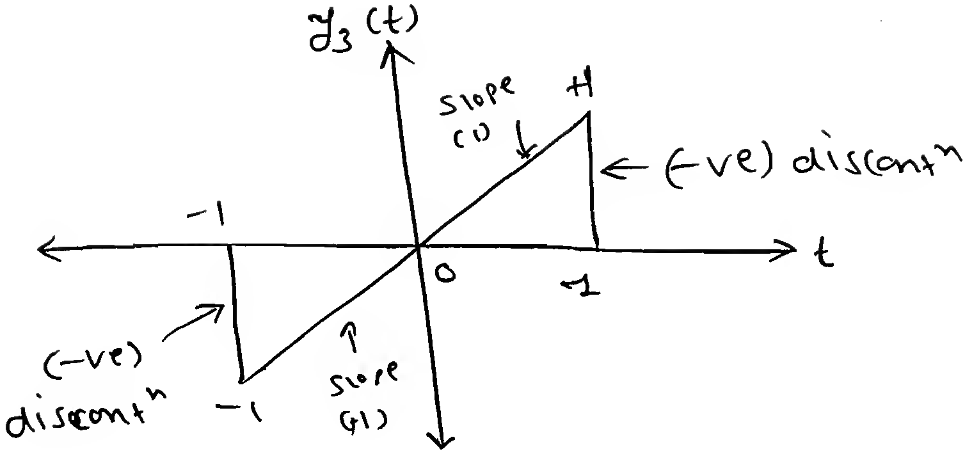
$\therefore \quad Y_3(\omega) = j \dfrac{d}{d\omega}\left(\dfrac{2\sin\omega}{\omega}\right)$

$= 2j\left[\dfrac{\omega\cos\omega - \sin\omega}{\omega^2}\right]$

$\therefore \quad \boxed{Y_3(\omega) = \dfrac{2j}{\omega^2}\left[\omega\cos\omega - \sin\omega\right]}$

Method-II : By differentiation.

$\Rightarrow$



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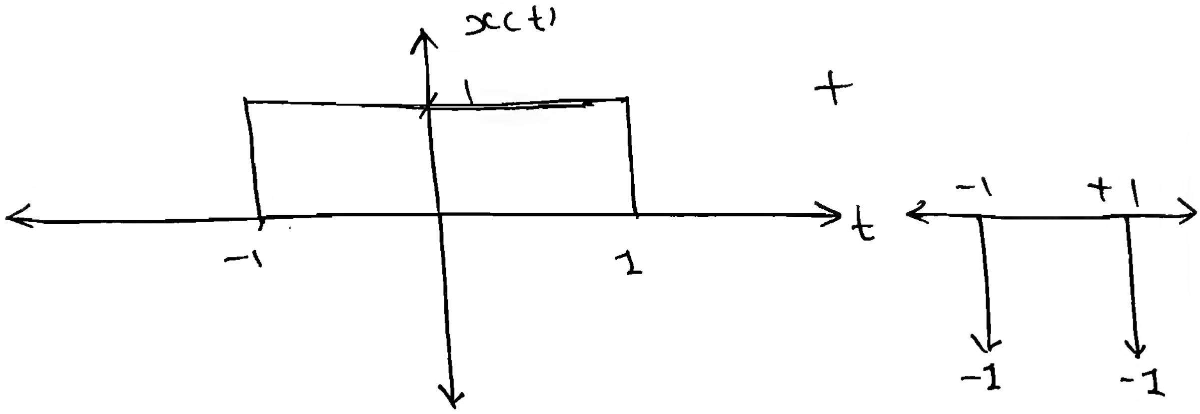


$$\therefore \quad \frac{dy_3(t)}{dt} = x(t) - \delta(t-1) - \delta(t+1).$$

$$\therefore \quad (j\omega)\, Y_3(\omega) = X(\omega) - \frac{e^{-j\omega} + e^{j\omega}}{1}.$$

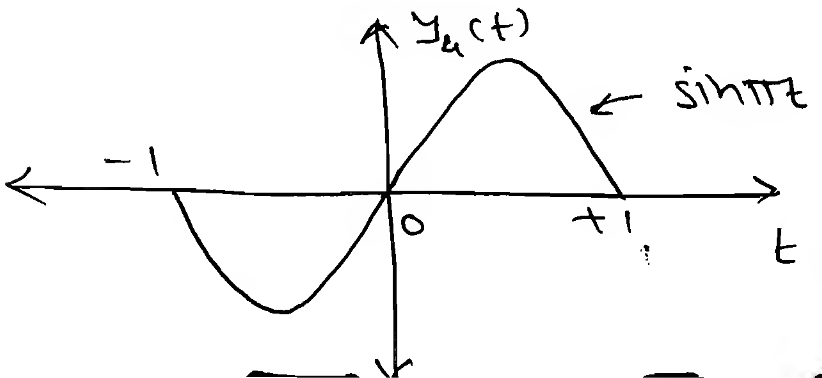
$$\therefore \quad Y_3(\omega) = \frac{X(\omega)}{j\omega} - \frac{2\cos\omega}{j\omega}.$$

$$= \frac{1}{j\omega}\left[\frac{2\sin\omega}{\omega} - 2\cos\omega\right]$$

$$= \frac{j}{j^2\omega^2}\left[2\sin\omega - 2\omega\cos\omega\right].$$

$$\boxed{Y_3(\omega) = \frac{2}{j\omega^2}\left[\omega\cos\omega - \sin\omega\right]}$$

(d)



Sol^n:

$$Y_4(t) = \sin \pi t \cdot x(t).$$

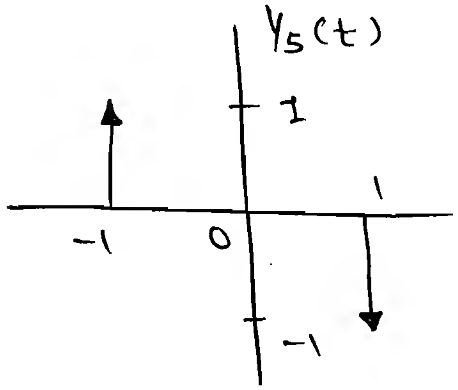
$$\therefore \cancel{Y_4(\omega) =} \; j \, \frac{d}{d\omega}$$

$$\therefore Y_4(t) = \frac{e^{j\pi t} - e^{-j\pi t}}{2j} \times x(t).$$

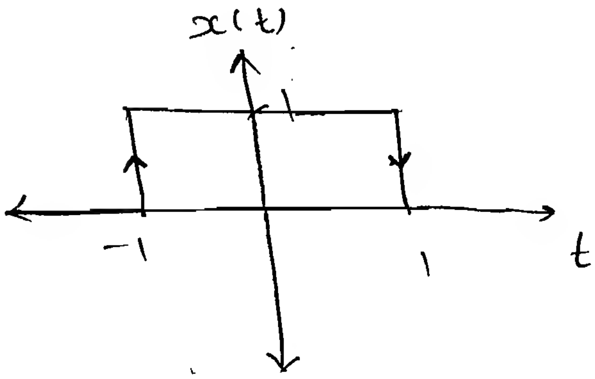
$$y_4(t) = \frac{1}{2j}\left[e^{+j\pi t} \cdot x(t) - e^{-j\pi t} \cdot x(t) \right].$$

$$\Rightarrow \boxed{Y(\omega) = \frac{1}{2j}\left[X(\omega - \pi) - X(\omega + \pi) \right]}$$

(e)



Sol^n:



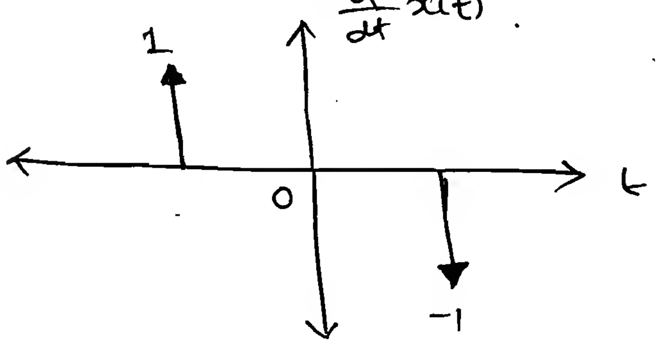
$$Y_5(t) = \frac{d}{dt} x(t).$$

$$\therefore Y_5(\omega) = j\omega \left[X(\omega) \right].$$

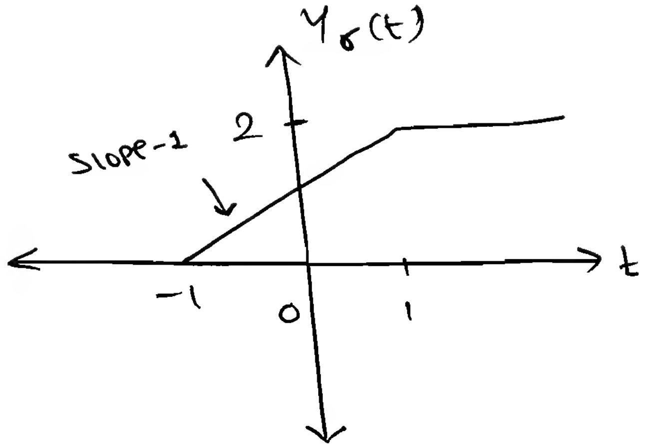
$$\therefore Y_5(\omega) = j\cancel{\omega} \left[\frac{2 \sin \omega}{\cancel{\omega}} \right)$$

$$\boxed{Y_5(\omega) = 2j \sin \omega}$$

$\Downarrow$ $\frac{d}{dt} x(t)$



(f)



Sol^n:

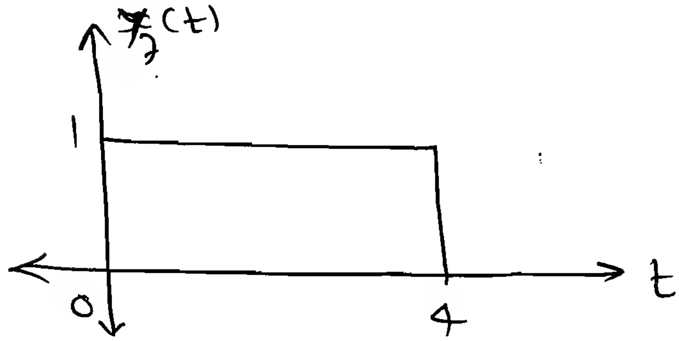
$$y_6(t) = (t+1)\,x(t) + 2u(t-1).$$

$$\therefore\ y_6(t) = t\,x(t) + x(t) + 2u(t-1).$$

$\downarrow$ F.T.

$$\therefore\ \boxed{Y_6(\omega) = j\frac{d}{d\omega}(X(\omega)) + X(\omega) + 2\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]e^{-j\omega}}.$$

(g)



Sol^n:

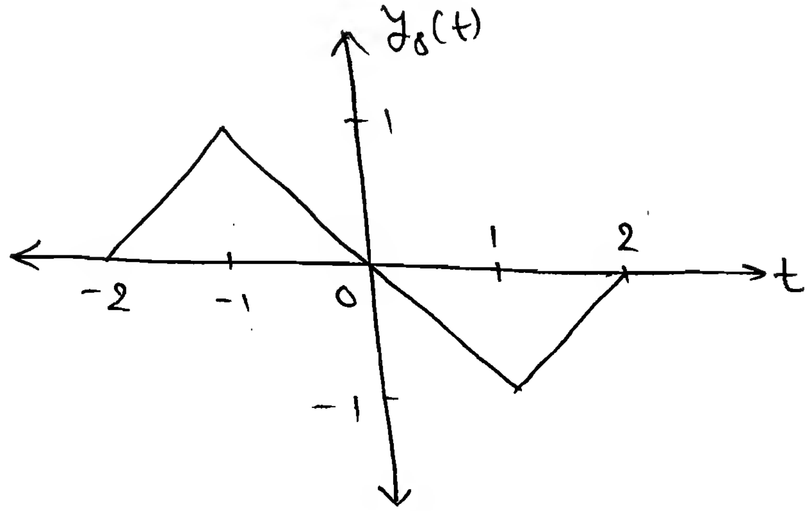
$$y_7(t) = x\left(\frac{t}{4} - 1\right).$$

$$y_7(t) = x\left(\frac{t-4}{4}\right).$$

$$\therefore\ Y_7(\omega) = \frac{1}{1/4}\cdot X\left(\omega/\tfrac{1}{4}\right)\cdot e^{-j4\omega}.$$

$$\therefore\ \boxed{Y_7(\omega) = 4\cdot e^{-j4\omega}\cdot X(4\omega).}$$

(h)



Soln:

$$y_8(t) = y_2(2(t+1)) - y_2(2(t-1)).$$

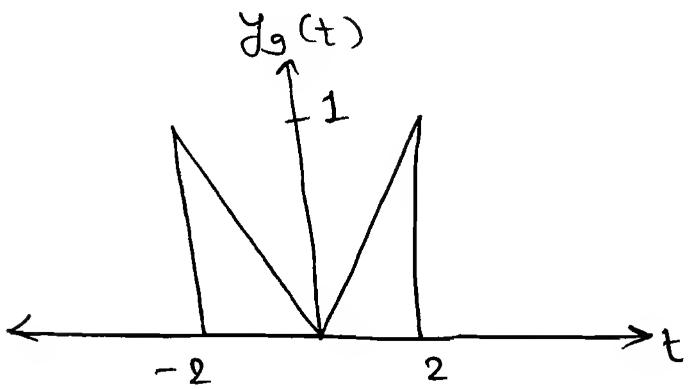
$$Y_8(\omega) = \frac{1}{(1/2)} \cdot e^{+j\omega} Y_2(\omega/\tfrac{1}{2}) - \frac{1}{(1/2)} \cdot e^{-j\omega} \cdot Y_2(\omega/\tfrac{1}{2}).$$

$$= 2e^{j\omega} Y_2(2\omega) - 2e^{-j\omega} Y_2(2\omega)$$

$$= 2 \times 2j \times Y_2(2\omega)\left[\frac{e^{j\omega} - e^{-j\omega}}{2j}\right].$$

$$\therefore \boxed{Y_8(\omega) = 4j\, Y_2(2\omega) \cdot \sin\omega.}$$

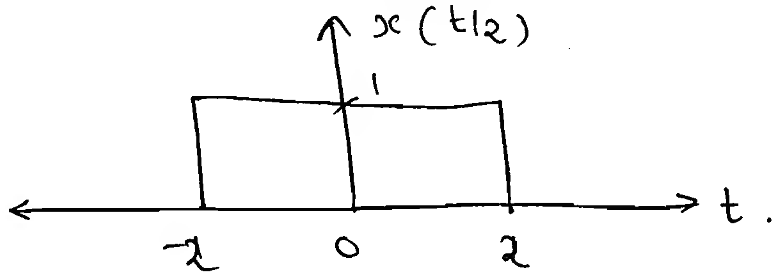
(i)



Soln:

$$y_9(t) = x(t/2) - y_2(t).$$

$\Rightarrow$



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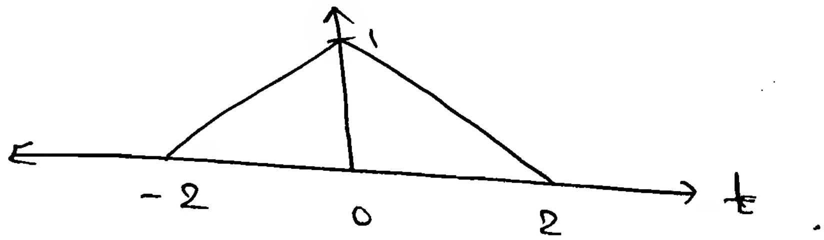

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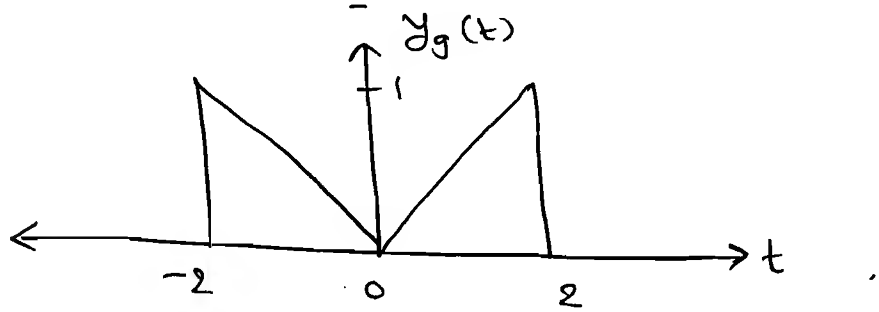
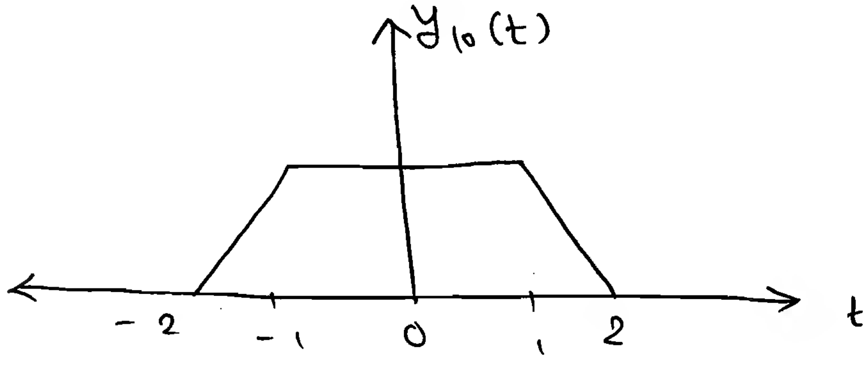


$\therefore \; y_9(t) = x(t/2) - y_2(t).$

$$\therefore \; \boxed{Y_9(\omega) = 2\,X(2\omega) - Y_2(\omega).}$$

***

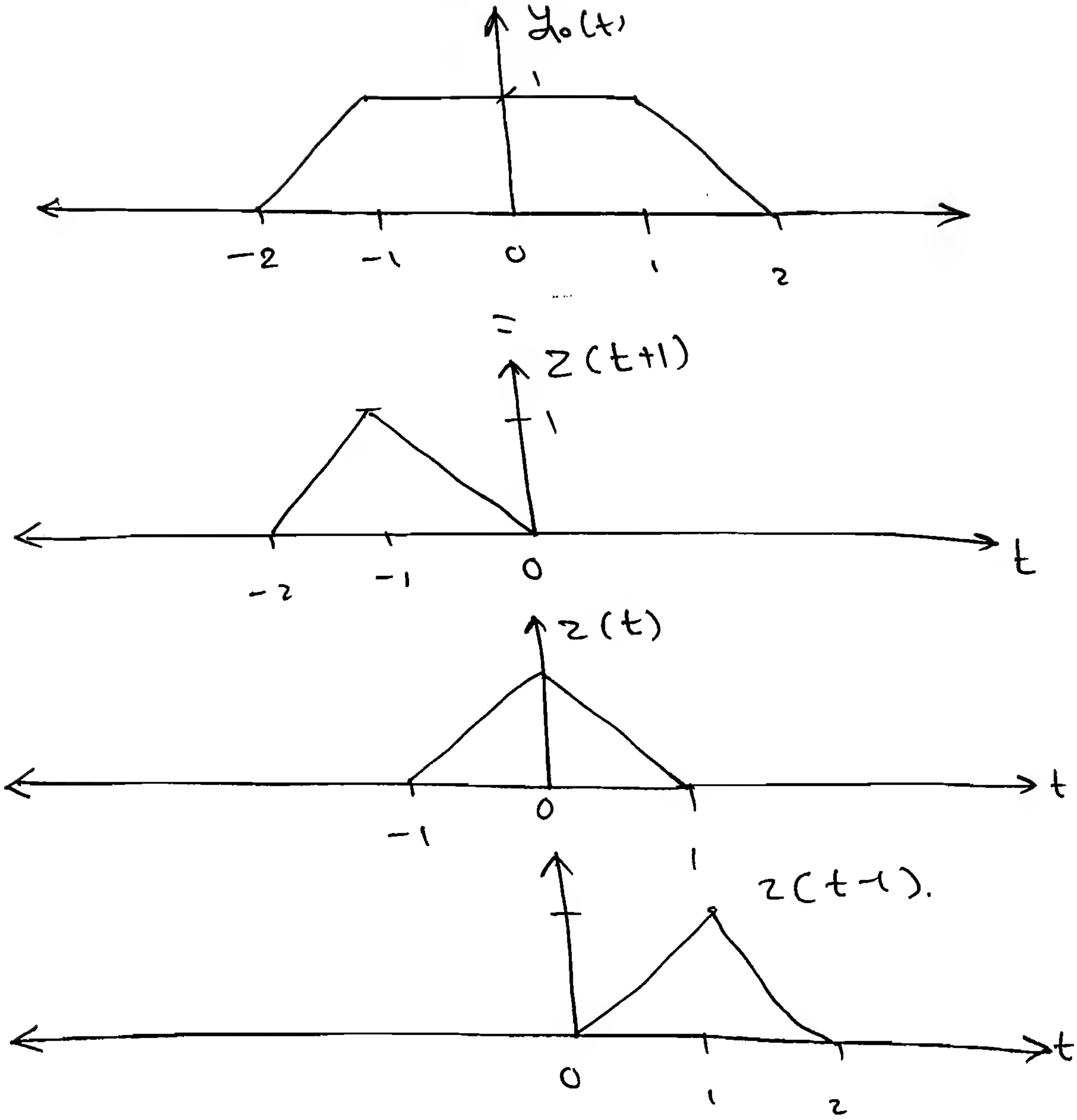
(j)



Solⁿ: let, $z(t) = \frac{1}{2}\, y_2(2t).$

$\therefore \; y_{10}(t) = z(t+1) + z(t) + z(t-1).$

$$\therefore \; y_{10}(t) = \frac{1}{2}\left[y_2(2t+2)\right] + \frac{1}{2}\left[y_2(2t)\right] + \frac{1}{2}\left[y_2(2t-2)\right].$$



$$\therefore \quad Y_{10}(\omega) = \frac{1}{2}\left[\frac{1}{2}e^{j\omega} Y_2(\omega/2) + \frac{1}{2}\cdot Y_2(\omega/2) + \frac{1}{2} Y_2(\omega/2)\cdot e^{-j\omega}\right].$$

$$\boxed{Y_{10}(\omega) = \frac{Y_2(\omega/2)}{4}\left[1 + 2\cos\omega\right].}$$

(7) Convolution in time :-

$\Rightarrow$ If $x(t) \longleftrightarrow X(\omega)$ & $h(t) \longleftrightarrow H(\omega)$.

then, $x(t) * h(t) \longleftrightarrow X(\omega)\cdot H(\omega)$.

⇒ Convolution in time corresponds to multiplication in frequency domain.

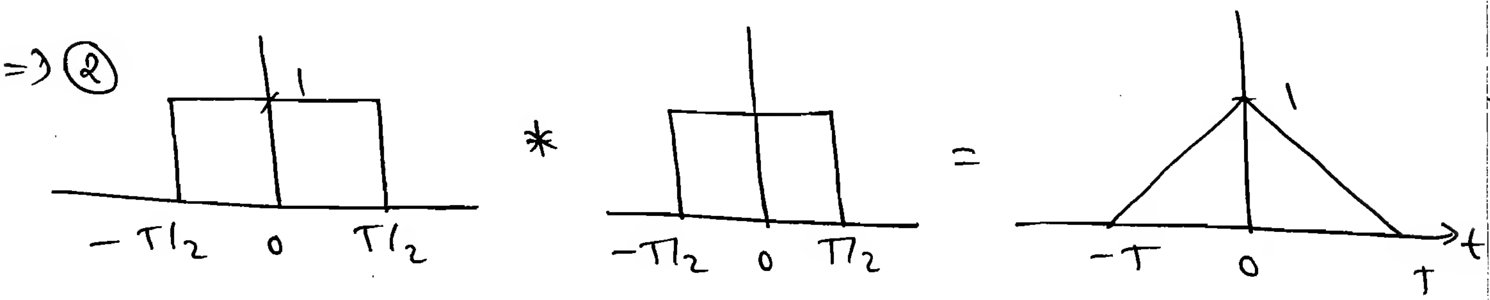
⇒ F.T. of of impulse response, $h(t)$ is known as frequency response, $H(\omega)$.

e.g.

① $e^{-at}\cdot u(t) * e^{-at}\cdot u(t) \xleftrightarrow{\text{F.T.}} \dfrac{1}{(a+j\omega)^2}$

$= t\cdot e^{-at}\cdot u(t)$

⇒ ②

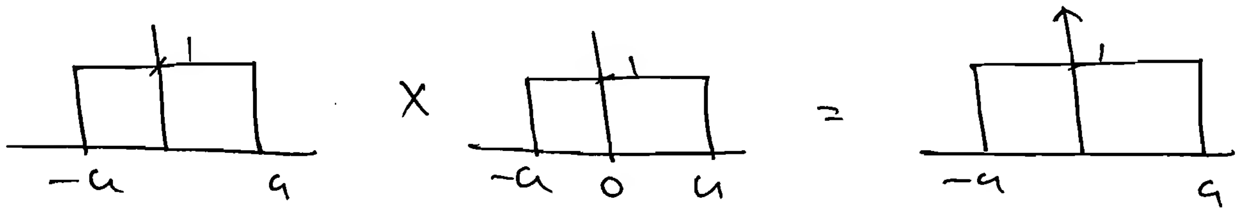


∴ $\text{rect}(t/T) * \text{rect}(t/T) \xleftrightarrow{\text{F.T.}} T^2\, Sa^2\left(\frac{\omega T}{2}\right).$

$T\,Sa\left(\frac{\omega T}{2}\right) \times T\,Sa\left(\frac{\omega T}{2}\right)$

③ $\boxed{\dfrac{\sin at}{\pi t} * \dfrac{\sin at}{\pi t} = \dfrac{\sin at}{\pi t}}$ I.F.T.

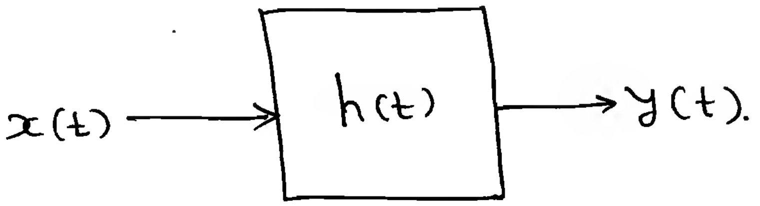
∴



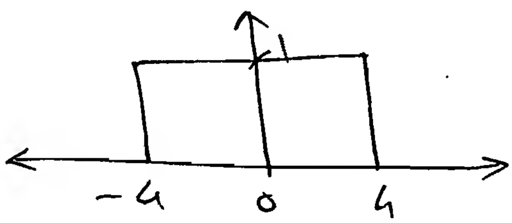
⇒ $\boxed{\text{Gaussian} * \text{Gaussian} = \text{Gaussian.}}$

P4.2.23 An L.T.I. system is having I.R. $h(t) = \frac{\sin 4t}{\pi t}$ for which the input applied is $x(t) = \cos 2t + \sin 6t$, find the o/p?

Soln:



$\therefore$ $H(\omega) = \text{rect}(t/8)$.



← Ideal LPF.

So, o/p = $\cos 2t$.

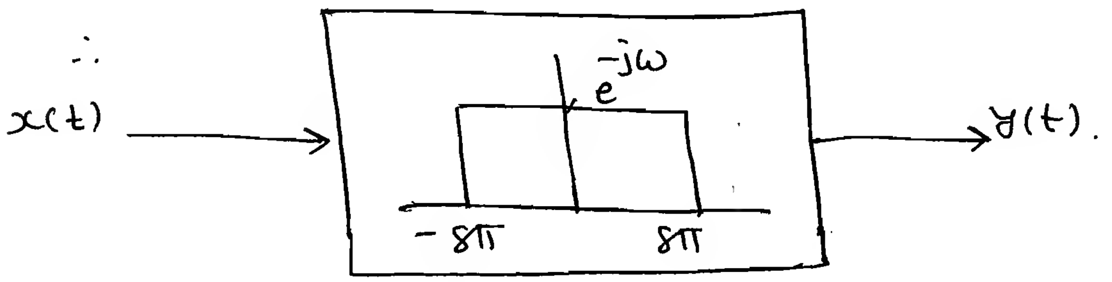
***

P4.2.26 (a) Find the o/p of a system having impulse response $h(t) = 8\,\text{sinc}[8(t-1)]$ when the input applied is $x(t) = \cos \pi t$.

Soln: $h(t) = 8\,\text{sinc}[8(t-1)]$.

$$h(t) = \frac{\cancel{8} \sin[8\pi(t-1)]}{\cancel{8}\pi(t-1)} \Rightarrow H(\omega) = e^{-j\omega} \cdot \text{rect}(t/16\pi)$$

$\therefore$



$\Rightarrow$ $\boxed{y(t) = \cos \pi(t-1)}$ ← delay because of $e^{-j\omega}$.

b) Let $g(t) = e^{-\pi t^2}$, and $h(t)$ is a filter matched to $g(t)$. If $g(t)$ is applied as input to $h(t)$, then the Fourier transform of the output is,

(a) $e^{-\pi f^2}$ (b) $e^{-\pi f^2/2}$ (c) $e^{-\pi|f|}$ (d) $e^{-2\pi f^2}$.

Soln:

$x(t) = e^{-\pi t^2}$ → Matched Filter $h(t) = x(T_0 - t)$. $= e^{-\pi t^2}$ →

$X(f) = e^{-\pi f^2}$

$Y(F) = X(f) \cdot H(f)$

$$\boxed{Y(f) = e^{-2\pi f^2}.}$$

$H(f) = e^{-\pi f^2}$

(delay is negated)

$\boxed{P\ 4.2.25}$ Using Convolution Property of F.T. find the Convolution of following signals.

(a) $y(t) = \text{rect}(t) * \cos(\pi t)$.

Soln: $Y(\omega) = X(\omega) \cdot H(\omega)$.

$$\therefore Y(\omega) = Sa\left(\frac{\omega T}{2}\right) \cdot \left[\pi\left[\delta(\omega - \pi) + \delta(\omega + \pi)\right]\right]$$

$$= \frac{\sin(\omega/2)}{(\omega/2)} \left[\pi\left[\delta(\omega - \pi) + \delta(\omega + \pi)\right]\right].$$

$$= \cancel{\tfrac{1}{\omega}} \cdot \cancel{\pi} \times \cancel{\frac{\sin(\pi/2)}{(\pi/2)}} + \cancel{\tfrac{1}{\omega}} \cdot \cancel{\pi} \cdot \cancel{\frac{\sin(-\pi/2)}{(-\pi/2)}}$$

$$\therefore\ Y(\omega) = \cancel{\pi} \times \frac{\cancel{\sin\frac{\pi}{2}}^{\,1}}{\cancel{\pi}/2}\,\delta(\omega-\pi)$$

$$+ \cancel{\pi} \times \frac{\cancel{\sin\left(\frac{-\pi}{2}\right)}^{\,-1}}{(-\cancel{\pi}/2)}\,.\,\delta(\omega+\pi).$$

$$\therefore\ Y(\omega) = 2\left[\delta(\omega-\pi) + \delta(\omega+\pi)\right].$$

$$\text{IFT}\downarrow \quad = \frac{2}{\pi}\left\{\pi\left[\delta(\omega-\pi) + \delta(\omega+\pi)\right]\right.$$

$$\therefore\ \boxed{Y(t) = \frac{2}{\pi}\cos\pi t}$$

(b) $y_3(t) = \cancel{\delta x(t) *}\ \text{rect}(t) * \cos(2\pi t).$

$$\Rightarrow\ y_3(t) = \text{rect}(t) * \cos(2\pi t).$$

$$= \frac{\sin\left(\frac{\omega}{2}\right)}{(\omega/2)} \times \left\{\pi\left[\delta(\omega-2\pi) + \delta(\omega+2\pi)\right]\right\}$$

$$= \frac{\pi\,.\,\cancel{\sin\left(\frac{2\pi}{2}\right)}^{\,0}}{\left(\frac{2\pi}{2}\right)}\,.\,\delta(\omega-2\pi)$$

$$+ \frac{\pi\sin\cancel{\left(\frac{-2\pi}{2}\right)}^{\,0}}{\left(-\frac{2\pi}{2}\right)}\,.\,\delta(\omega+2\pi)$$

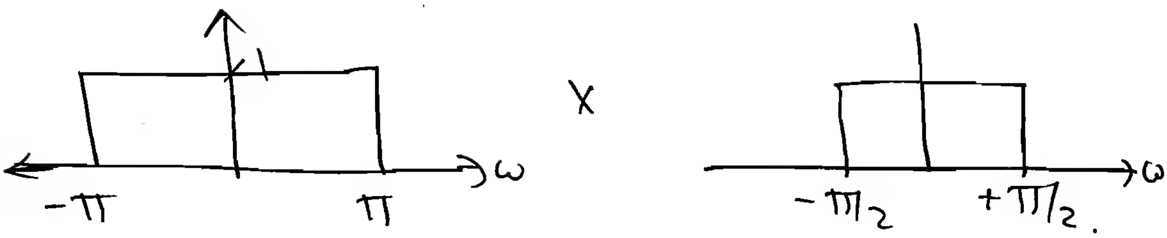
$$= 0 + 0$$

$$\boxed{y_3(t) = 0}$$

(c) $y_3(t) = \text{sinc}(t/2) * \sin(t/2).$

Sol^n: $y_3(t) = \text{sinc}(t) * \text{sinc}(t/2)$.

$Y_3(\omega) = X_1(\omega) \cdot X_2(\omega)$.



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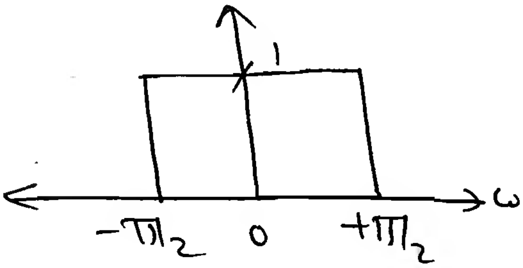
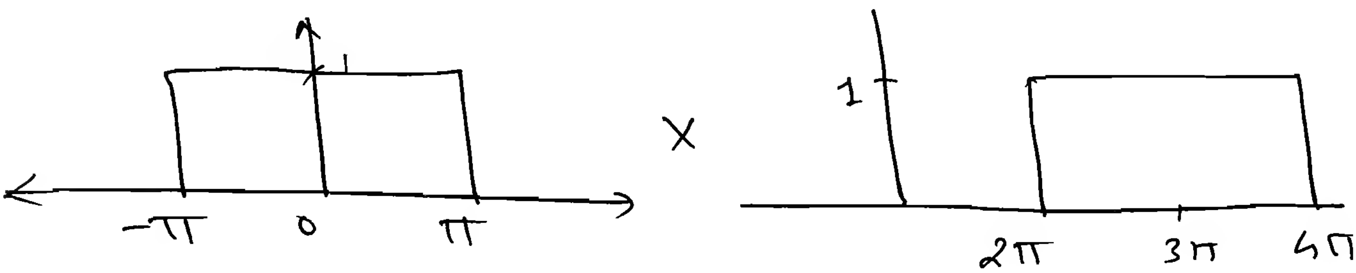


$$Y_3(\omega) = \left[\frac{\sin(\pi t)}{(\pi t)}\right] * \left[\frac{\sin\left(\frac{\pi t}{2}\right)}{\left(\frac{\pi t}{2}\right)}\right].$$

$$\boxed{y_3(t) = \text{sinc}(t/2).}$$

(d) $y_4(t) = \text{sinc}(t) * e^{j3\pi t} \cdot \text{sinc}(t)$.

Sol^n: $y_4(t) = \left[\frac{\sin \pi t}{\pi t}\right] * \left[e^{j3\pi t} \cdot \frac{\sin \pi t}{\pi t}\right]$.

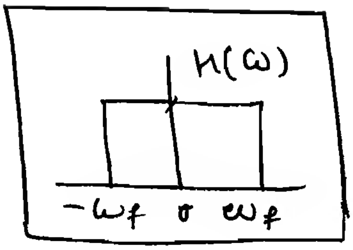


$= 0$.

So, $\boxed{y_4(t) = 0}$

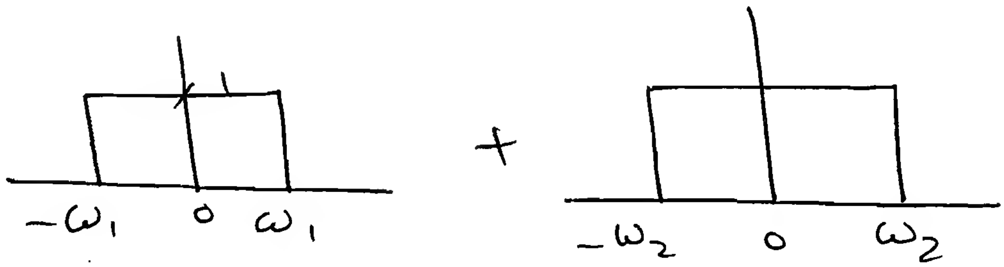
Q

I/P: $\frac{\sin \omega_1 t}{\pi t} + \frac{\sin \omega_2 t}{\pi t}$



$\longrightarrow$ O/P = ?

⇒ ① $\omega_f < \omega_1$



let, $\omega_f = 0.5$

$$o/p = \frac{\sin \omega_f t}{\pi t} + \frac{\sin \omega_f t}{\pi t}.$$

$$o/p = \frac{2 \sin \omega_f t}{\pi t}$$

② $\omega_1 < \omega_f < \omega_2$.

let, $\omega_f = 1.5$

$1 < 1.5 < 2.$

$$o/p = \frac{\sin \omega_1 t}{\pi t} + \frac{\sin \omega_f t}{\pi t}.$$

③ $\omega_f > \omega_2$; $\omega_f = 2.5$.

$\therefore$ $\boxed{O/P = I/P}$

$\boxed{P\,4.2.22}$ Given $y(t) = x(t) * h(t)$ and $g(t) = x(3t) * h(3t)$ such that $g(t) = A\,y(Bt)$, find A & B?

$\underline{\underline{\text{Sol}^n}}$: $y(t) = x(t) * h(t)$.

$$\Rightarrow \quad Y(\omega) = X(\omega) \cdot H(\omega).$$

$$\longrightarrow \quad g(t) = x(3t) * h(3t).$$

$$G(\omega) = \frac{1}{3} X\left(\frac{\omega}{3}\right) \cdot \frac{1}{3} H\left(\frac{\omega}{3}\right).$$

$$G(\omega) = \frac{1}{9} H\left(\frac{\omega}{3}\right) \cdot X(\omega/3). \quad \text{— } 1$$

$$\rightarrow \quad g(t) = A\,y(Bt).$$

$$G(\omega) = \frac{A}{B} Y(\omega/B). \quad \text{— } 2$$

Compare eqn 1 & 2.

$$\frac{A}{B} = \frac{1}{9}, \quad \& \quad \cancel{B \neq 1/B}. \quad \cancel{B \neq 1/3}$$

$$\boxed{B = 3}$$

$$\Rightarrow \quad A = 3/9$$

$$\boxed{A = 1/3}$$

$\boxed{P\,4.2.24}$ Let $x(t)$ be a signal whose F.T. is $X(\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5)$ & Let $h(t) = u(t) - u(t-2)$.

(a) is $x(t)$ periodic?

(b) is $x(t) * h(t)$ $h(t)$ is periodic?

Soln: (a)

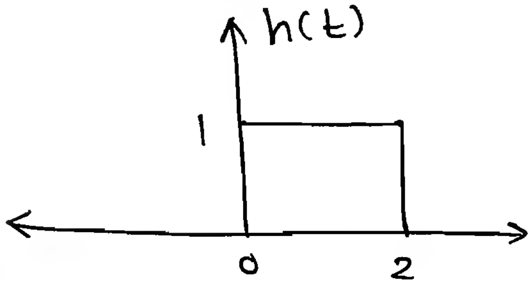
$$X(\omega) = \delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5).$$

$$x(t) = \frac{1}{2\pi} + \frac{e^{-j\pi t}}{2\pi} \cdot 1 + \frac{1}{2\pi} e^{-j5t} \cdot 1.$$

So, G.C.D $(\pi, 5) \not b$

So, Not periodic signal.

(b) $h(t) = u(t) - u(t-2)$



$$\xleftrightarrow{F.T.} e^{-j\omega} \cdot 2Sa\left(\frac{\omega \cdot 2}{2}\right)$$

$$= e^{-j\omega} \frac{2\sin\omega}{\omega}.$$

$$= \frac{2\sin\omega}{\omega} \cdot e^{-j\omega}.$$

$$\therefore Y(\omega) = X(\omega) \cdot H(\omega)$$

$$\therefore Y(\omega) = \left[\delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)\right] \times \left[\frac{2\sin\omega}{\omega} \cdot e^{-j\omega}\right].$$

$$= \frac{e^{-j\omega}}{\omega} \cdot 2\sin\omega \cdot \delta(\omega) + \frac{e^{-j\omega}}{\omega} \cdot 2\sin\omega\, \delta(\omega-\pi).$$

$$+ \frac{e^{-j\omega}}{\omega} \cdot 2\sin\omega \cdot \delta(\omega-5).$$

$$Y(\omega) = 2\delta(\omega) + 0 + \frac{1}{\omega} 2\sin 5 \cdot \delta(\omega-5).$$

$$\therefore \boxed{Y(\omega) = 2\delta(\omega) + \frac{1}{\omega} \cdot 2\sin 5\, \delta(\omega-5).}$$

So, o/p is periodic. ($\because$ GCD $(0,5) = 5$).

(8) Frequency Convolution:

$\Rightarrow$ If $x_1(t) \longleftrightarrow X_1(\omega)$ and

$x_2(t) \longleftrightarrow X_1(\omega)$ then.

$$\boxed{x_1(t)\cdot x_2(t) \xleftrightarrow{F.T.} \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right].}$$

*** Application:**

1. Modulator
2. Samples in freq. domain.

e.g.: $x(t)\cdot \cos\omega_c t \xleftrightarrow{F.T.} \frac{1}{2\cancel{\pi}}\left[X(\omega) * \cancel{\pi}\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]\right]$

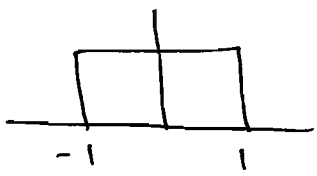
$$= \frac{X(\omega-\omega_c) + X(\omega+\omega_c)}{2}.$$

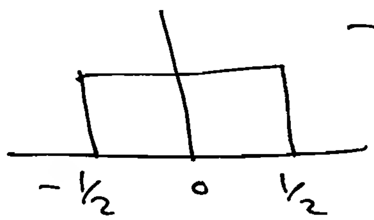
$\boxed{P\ 4.2.27}$ Find the F.T. of

(b) $x(t) = \dfrac{\sin t \cdot \sin(t/2)}{\pi t^2}.$

Soln: $x(t) = \dfrac{\sin t}{\pi t} \times \dfrac{\sin(t/2)}{\pi t} \times \pi$

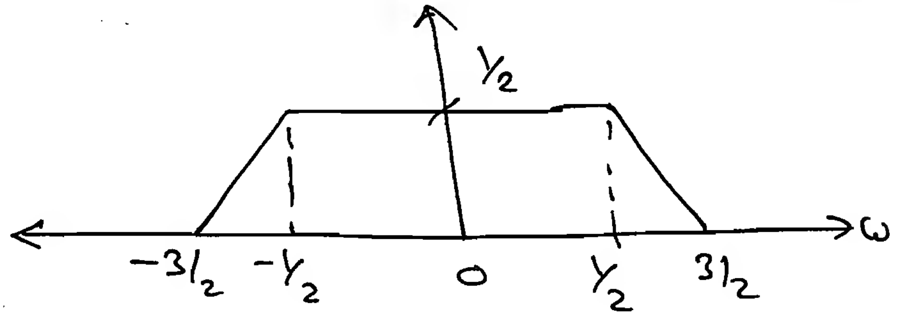
$\therefore\ X(\omega) = \frac{1}{2\cancel{\pi}} \times \cancel{\pi}\left[\text{rect}(\omega/2) * \text{rect}(\omega).\right].$

$X(\omega) = \frac{1}{2}\Big[$ 

$*$ 

$\Big]$

$\therefore \quad X(\omega) =$



(g) Integration in Time:

$\Rightarrow$ If $x(t) \longleftrightarrow X(\omega)$

then $\boxed{\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\,\delta(\omega).}$

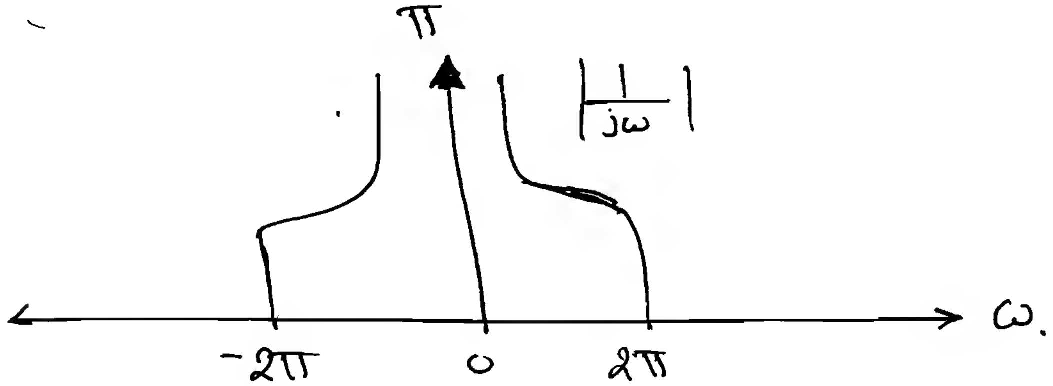
P. 4.2.28 Find the F.T. of $\int_{-\infty}^{t} \frac{\sin 2\pi t}{\pi t}\,dt$.

Soln:

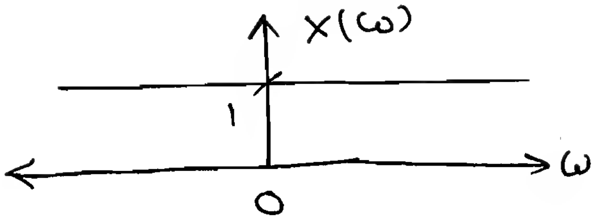
$$x(t) = \int_{-\infty}^{t} \frac{\sin 2\pi t}{\pi t}\,dt$$

$$X(\omega) = \frac{X_1(\omega)}{j\omega} + \pi X(0)\,\delta(\omega).$$

$$X(\omega) = \frac{\text{rect}\,(\omega/4\pi)}{j\omega} + \pi(1).\delta(\omega).$$



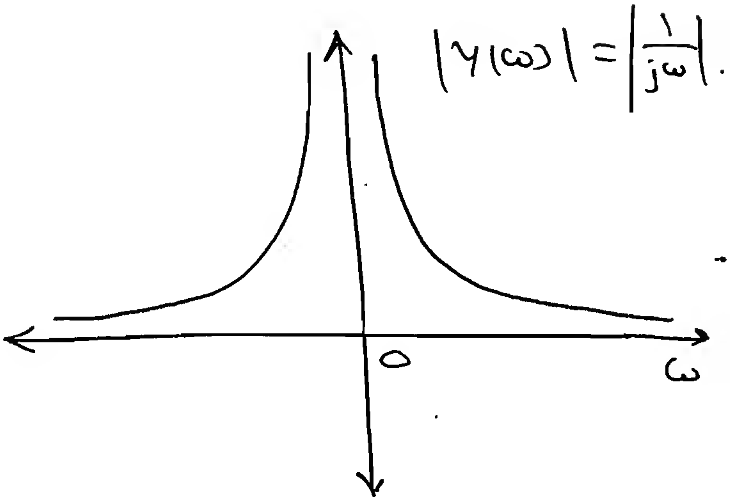
$\Rightarrow$ $x(t) = \delta(t) \longleftrightarrow$



$$\underbrace{\int_{-\infty}^{t} \delta(\tau)\,d\tau}_{= u(t).} \longleftrightarrow \underbrace{\frac{1}{j\omega} + \pi(1)\,\delta(\omega).}_{Y(\omega)}$$

i.e. $u(t) \xleftrightarrow{F.T.} \dfrac{1}{j\omega} + \pi\,\delta(\omega).$

$$|Y(\omega)| = \left|\frac{1}{j\omega}\right|.$$



$\Rightarrow$ Differentiation gives only $\frac{1}{j\omega}$ terms, $\pi\delta(\omega)$ [DC term] is missing there.

☆ Rayleigh's Energy theorem (or)
Parseval's Power theorem :-

$\Rightarrow$ Area under spectral density represents energy (or) power in the signal.

$\Rightarrow$ When a signal is added with noise by comparing the noise spectral density with signal spectral density we can suppress the noise component by

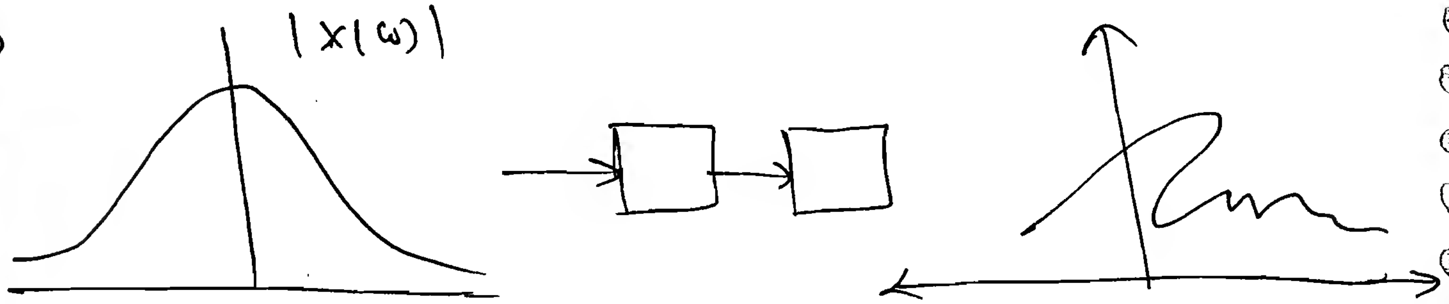
designing an 'adaptive Filter'. [Adjustable coefficient].

$\Rightarrow$ IR $\rightarrow$ LTI filter.

Adaptive filter $\longrightarrow$ LTI filter.

$\Rightarrow$ $$\boxed{\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2 . d\omega .}$$

$\Rightarrow$



$\Rightarrow$

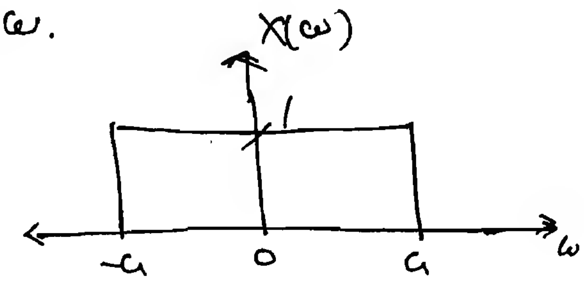
P 4.2.29 Find the energy in the signal

$x(t) = \frac{\sin at}{\pi t}$.

Soln:

$$E_{x(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega .$$

$x(t) = \frac{\sin at}{\pi t} \xleftrightarrow{F.T}$

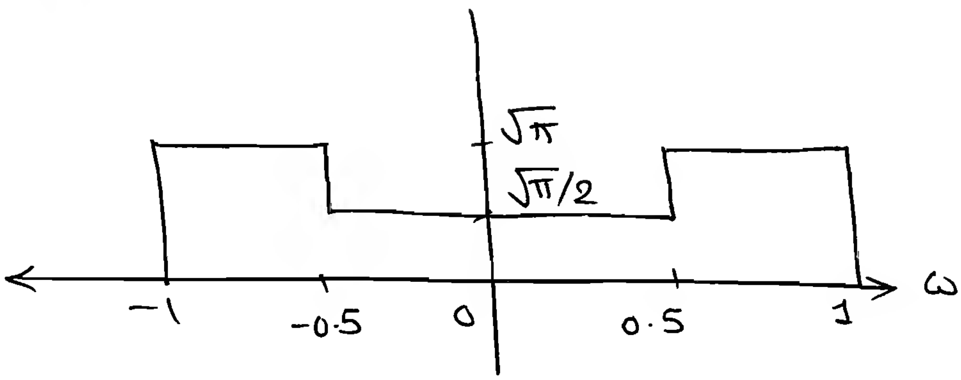


$$\therefore \quad E_{x(t)} = \frac{1}{2\pi} \int_{-a}^{a} (1)^2 \, d\omega.$$

$$= \frac{2a}{2\pi}$$

$$\therefore \boxed{E_{x(t)} = \frac{a}{\pi}}$$

$\boxed{\text{P 4.2. 30}}$ Find the energy in the Speltrum shown in fig?



Soln:

$$E_{x(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(\omega)|^2 \, d\omega.$$

$$= \frac{1}{2\pi} \times 2 \times \int_{0}^{1} |x(\omega)|^2 \, d\omega$$

$$= \frac{1}{\pi} \times \left[\int_{0}^{0.5} \left(\frac{\sqrt{\pi}}{2}\right)^2 d\omega + \int_{0.5}^{1} (\sqrt{\pi})^2 \, d\omega \right].$$

$$= \frac{1}{\pi} \left[\frac{\pi}{4} \times \frac{1}{2} + \frac{\pi}{2} \right].$$

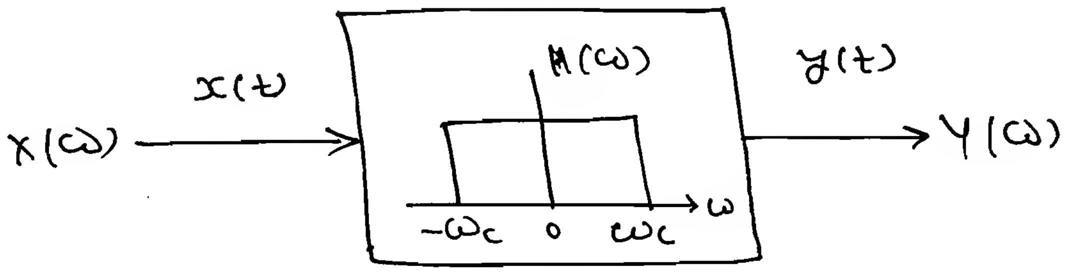
$$= \frac{1}{8} + 1/2$$

$$\boxed{E_{x(t)} = 5/8}$$

$\boxed{P4.3.21}$ An input signal $x(t) = e^{-2t}u(t)$ is applied to an ideal L.P.F. with freq. response char.
$$H(\omega) = \begin{cases} 1 \; ; & |\omega| < \omega_c . \\ 0 \; ; & |\omega| > \omega_c . \end{cases}$$

Find ω_c, such that energy in the

Soln:



→ i/p : $x(t) = e^{-2t} \cdot u(t).$

$$X(\omega) = \frac{1}{2+j\omega}.$$

$$|X(\omega)|^2 = \frac{1}{4+\omega^2}.$$

→ $$E_{x(t)} = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{0}^{\infty} e^{-4t} \, dt.$$

$$\boxed{E_{x(t)} = 1/4}$$

Now, given that

$$E_{y(t)} = \frac{1}{2} \times E_{x(t)}.$$

$$\therefore \; E_{y(t)} = 1/8 .$$

Now, $Y(\omega) = X(\omega) \cdot H(\omega)$.

$$Y(\omega) = \frac{1}{\sqrt{\omega^2+4}} \quad (1).$$

$$\therefore E_{y(t)} = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{1}{\omega^2+4} \cdot d\omega.$$

$$= \frac{2}{2\pi} \times \int_{0}^{\omega_c} \frac{1}{\omega^2+4} \cdot d\omega.$$

$$\therefore \frac{1}{8} = \frac{1}{\pi} \times \frac{1}{2} \times \left[\tan^{-1}(\omega/2)\right]_0^{\omega_c}$$

$$\therefore \frac{\pi}{4} = \tan^{-1}(\omega_c/2).$$

$$\therefore \omega_c = 2\tan\frac{\pi}{4}.$$

$$\therefore \boxed{\omega_c = 2 \text{ rad/sec.}}$$

$\boxed{\text{P4.2.32}}$ Consider $x(t) \longleftrightarrow X(\omega)$. Suppose we are given the following facts

(i) $x(t)$ is real and non negative.

(ii) $F^{-1}\{(1+j\omega)X(\omega)\} = A \cdot e^{-2t} u(t)$, where 'A' is independent of t.

(iii) $\int_{-\infty}^{+\infty} |X(\omega)|^2 \cdot d\omega = 2\pi$ find a closed-form expression for $x(t)$.

Soln:

$$F^{-1}\{(1+j\omega)\,X(\omega)\} = A\,e^{-2t}.u(t).$$

$$\therefore\ (1+j\omega)\,X(\omega) = F\{e^{-2t}.\,u(t)\}.$$

$$\therefore\ (1+j\omega)\,X(\omega) = \frac{1}{2+j\omega}.$$

$$\therefore\ X(\omega) = \frac{1}{(1+j\omega)(2+j\omega)}.$$

$$\therefore\ X(\omega) = A\left[\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right]$$

$$\therefore\ x(t) = A\left[e^{-t} - e^{-2t}\right]u(t).$$

Now, $$\int_{-\infty}^{+\infty} |X(\omega)|^2\,d\omega = 2\pi$$

$$\Rightarrow\ \boxed{E_{x(t)} = 1} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2\,d\omega.$$

$$\therefore\ 1 = \int_{-\infty}^{\infty} |x(t)|^2\,dt.$$

$$\therefore\ 1 = A^2\left[\int_0^{\infty}\left[e^{-2t} - 2e^{-3t} + e^{-4t}\right]dt\right].$$

$$\therefore\ 1 = A^2\left[\left[\frac{e^{-2t}}{-2} - \frac{2}{-3}e^{-3t} + \frac{e^{-4t}}{-4}\right]_0^{\infty}\right].$$

$$\therefore\ 1 = A^2\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right].$$

$$\therefore\ \cancel{\frac{6}{6}}\ \cancel{2A^2}\quad 1 = A^2\left[\frac{6-8+3}{12}\right].$$

$$\Rightarrow \boxed{A = \sqrt{12}.}$$

$$\therefore \boxed{x(t) = \sqrt{12}\left[e^{-t} - e^{-2t}\right]u(t).}$$

$\boxed{P\ 4.2.33}$ Find the value of the integral

$$\int_{-\infty}^{+\infty} \frac{8}{(\omega^2+4)^2}\, d\omega.$$

Soln:

$$\int_{-\infty}^{+\infty} \frac{8}{(\omega^2+4)^2}\, . d\omega$$

$$\rightarrow e^{-\alpha|t|} \longleftrightarrow \frac{2\alpha}{\omega^2+\alpha^2}.$$

$$\Rightarrow \frac{1}{2}\int_{-\infty}^{+\infty} \left(\frac{2(2)}{\omega^2+\alpha^2}\right)^2 . d\omega.$$

$$= \frac{1}{2} \times 2\pi \times \int_{-\infty}^{\infty} e^{-4|t|}\, . dt.$$

$$= \pi \times 2 \times \int_{0}^{\infty} e^{-4t}\, . dt.$$

$$= 2\pi \times \frac{1}{4}.$$

$$= \frac{\pi}{2}.$$

$$\therefore \text{So, } \int_{-\infty}^{+\infty} \frac{8}{(\omega^2+4)^2}\, d\omega = \pi/2.$$

a) Using Parseval's theorem find the signal energy in $x(t) = 4\,\text{sinc}(t/5)$.

Soln:

$$x(t) = 4\,\text{sinc}(t/5).$$

$$A = 4, \quad T = 5.$$

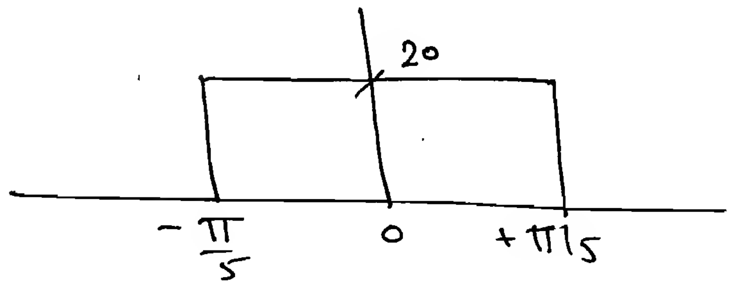
$$\therefore \; X(\omega) = AT\,\text{rect}(\omega/2a).$$

$$x(t) = \frac{4\sin(\pi t/5)}{\pi t/5}$$

$$x(t) = 20\,\frac{\sin\left(\frac{\pi t}{5}\right)}{\pi t}$$

$\downarrow$ F.T.

$$X(\omega) = 20\,.\,\text{rect}\left(\omega/2\left(\tfrac{\pi}{5}\right)\right).$$



$$\therefore \; E_{x(t)} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2 . d\omega.$$

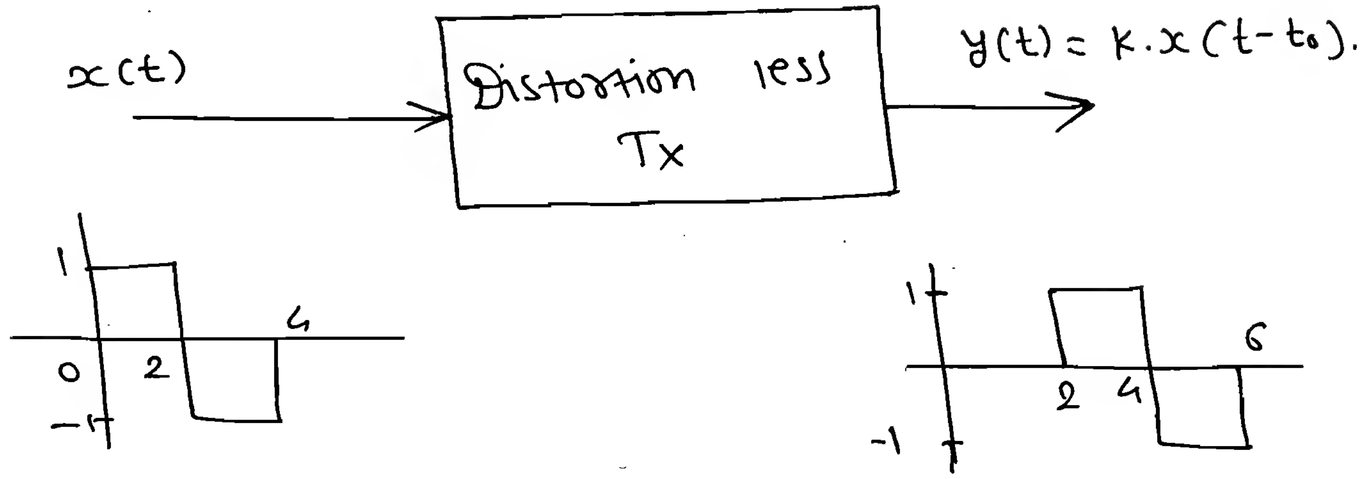
$$= \frac{1}{2\pi} \times 2 \times \int_{0}^{\pi/5} (400)\,.d\omega.$$

$$= \frac{1}{\cancel{\pi}} \times \cancel{400}^{80} \times \frac{\cancel{\pi}}{\cancel{5}}$$

$$\therefore \; \boxed{E_{x(t)} = 80}$$

* Applications:

① Distortionless Transmission:-



⇒ For distortionless transmission, output is replica of the input which scaling in its amplitude and possible delay.

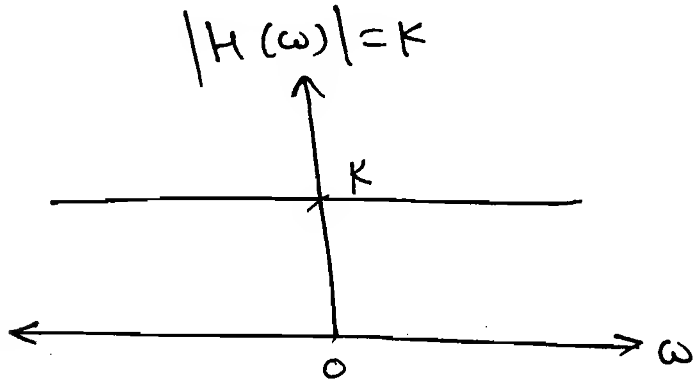
⇒ For Distortion less transmission, magnitude response must be a constant, phase response must be linear function of ω with slope $-t_0$, where t_0 is delay in output with respect to input.

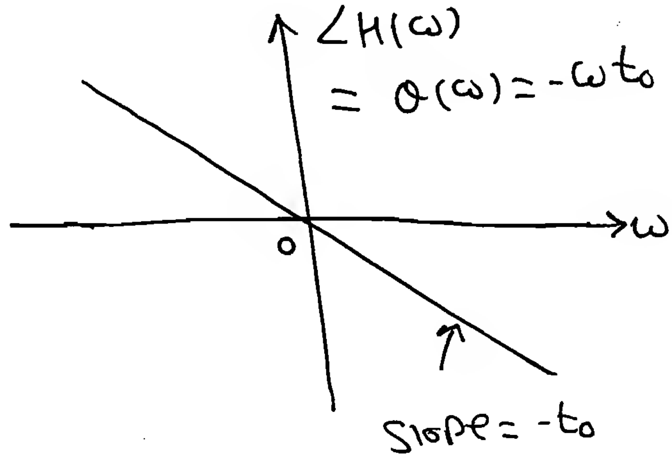
$$y(t) = K\,x(t - t_0).$$

$$\therefore\ Y(\omega) = K \cdot e^{-j\omega t_0} \cdot X(\omega).$$

$$\therefore\ Y(\omega) = |H(\omega)| \cdot e^{j\theta(\omega)}$$

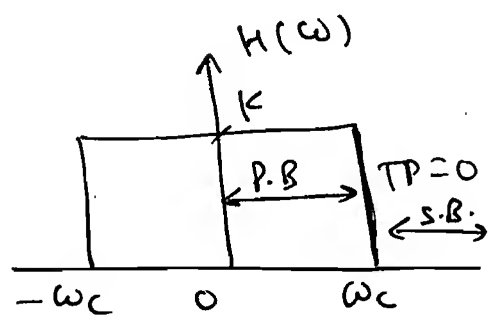
$$\Rightarrow\ |H(\omega)| = K, \quad \boxed{\theta(\omega) = -\omega t_0}$$



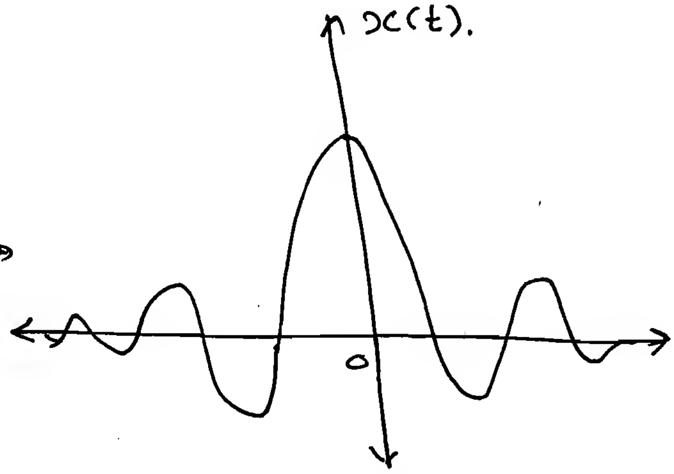


⇒ According to Rayleigh's theorem for a distortionless condition we require infinite energy which is impractical so we are limitting the range of freq. from 0 to ω_c i.e. Ideal filter (IFT of rectangular spectrum is a Sa f^n which extends for all time.).

⇒ All ideal filters are non-causal and unstable.



IFT



⇒ Transition width is deciding the order of the filter (no. of energy storing elements).

⇒ Most of the Practical Systems we are designing as non-linear phase response. To make it as linear we are defining two parameters.

① **Phase delay:**

⇒ It is the delay i.e. occuring at a single freq. which is due to carrier

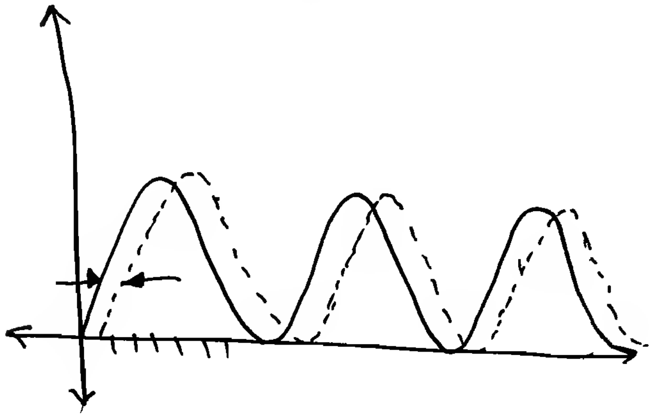
$$\boxed{t_p(\omega) = -\frac{\theta(\omega)}{\omega}}$$

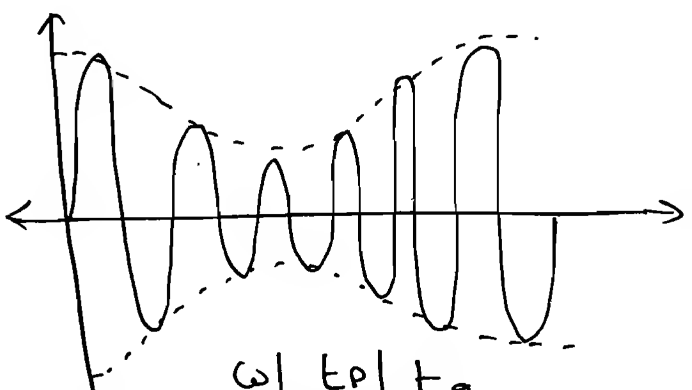
② **Group delay:**

⇒ It is the delay i.e. occuring at a group (or) narrow band of freq. which is due to envelope of the msg signal.

$$\boxed{t_g(\omega) = -\frac{d\theta(\omega)}{d\omega}}$$

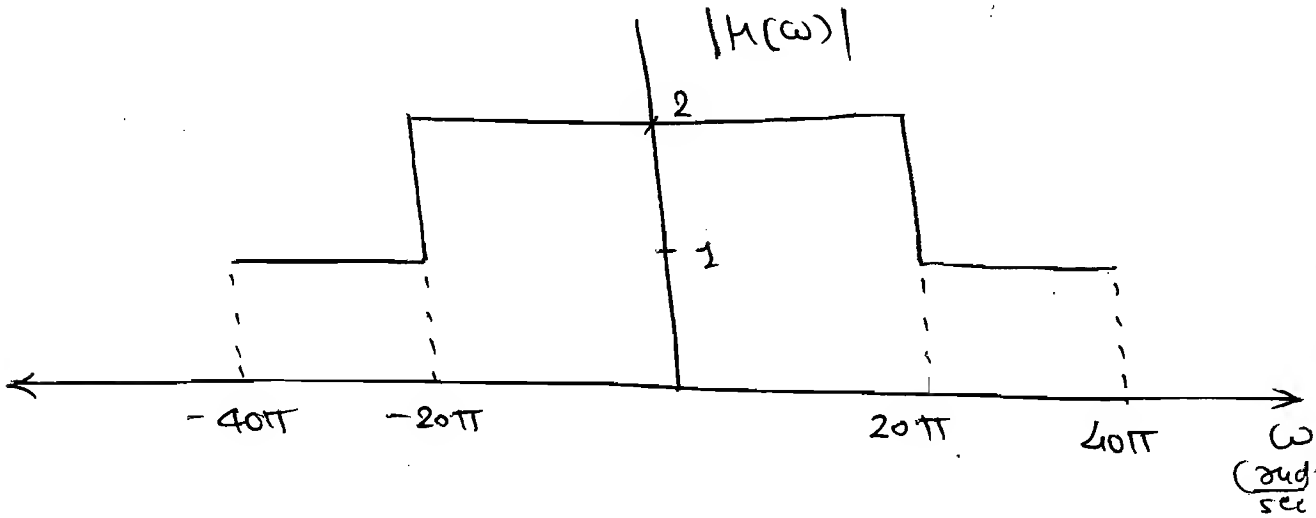
⇒ '−' indicates the phase lag present in the ~~[illegible]~~ system.

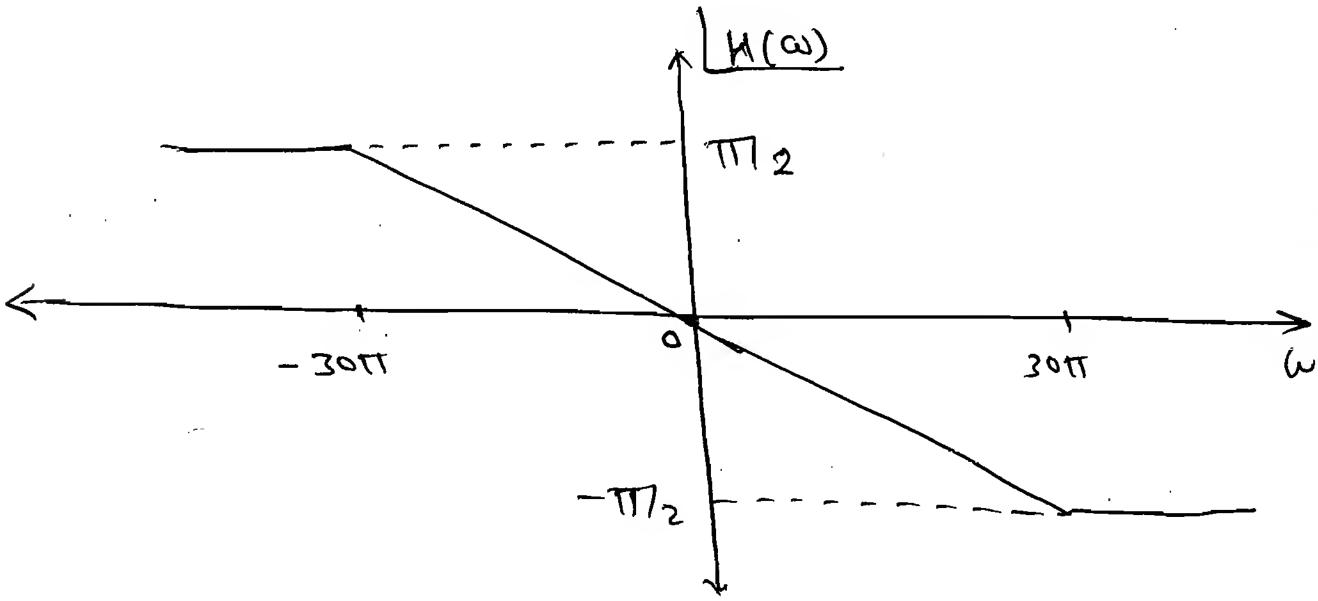




ω	t_p	t_g

P4.3-1 Consider a transmission system $H(\omega)$ with magnitude and phase response as shown in figure. If an input $x(t) = 2\cos 10\pi t + \sin 26\pi t$ is given to the system the output will be ___.





Sol^n:

$x(t) = 2\cos 10\pi t + \sin 26\pi t.$

① $\cos 10\pi t$

at $\pm 30\pi$ $M = 1$

$\phi \Rightarrow \pm \pi/2$

So, 30π $-\pi/2$

10π $(?)$

$\Rightarrow \phi = -\pi/2 \times 1/3 = -\pi/6.$

So, at 10π, $m \Rightarrow 2$

$\phi \rightarrow -\pi/6$.

② $\sin 26\pi t$.

m at $26\pi \Rightarrow \cancel{2}\ 1$.

$\phi \Rightarrow$ $30\pi \rightarrow \pi/2$

$26\pi \rightarrow (?)$

$$\Rightarrow \phi = -\frac{\overset{13}{\cancel{26\pi}}}{\cancel{30\pi}} \times \pi/\cancel{2}$$

$$\phi = -\frac{13\pi}{30}.$$

So, Ans: $(2)(2)\cos\left(10\pi t - \pi/6\right)$.

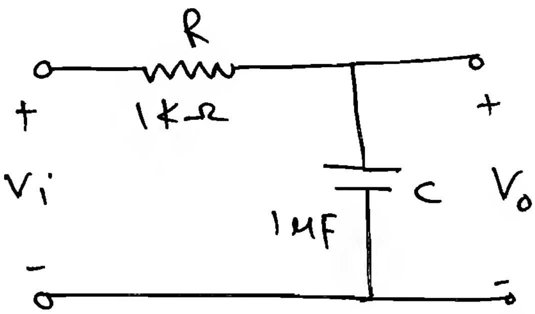
$+ \sin\left(30\pi t - \frac{13\pi}{30}\right)$.

$$\boxed{y(t) = 4\cos(10\pi t - \pi/6) + \sin\left(26\pi t - \frac{13\pi}{30}\right).}$$

Q The system under consideration is an RC LPF with $R = 1\,k\Omega$ & $C = 1\,\mu F$

a) Let $H(f)$ denote the frequency response of an RC LPF with let f_1 be the highest freq. component such that $0 \le |f| \le f_1$, $\left|\frac{H(f_1)}{H(0)}\right| \geq 0.95$ then f_1 (in Hz) is ______.

Soln:



$$H(f) = \frac{1}{1 + j2\pi fRC}.$$

$$|H(f)| = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$$

Now, given that

$$|H(f_1)| \geq 0.95\,|H(0)|.$$

So, $$\frac{1}{\sqrt{1+(2\pi f_1 RC)^2}} \geq 0.95 \times \frac{1}{\sqrt{1+0}}.$$

$$\Rightarrow \quad \frac{1}{1+(2\pi f_1 RC)^2} = 0.9025.$$

$$\Rightarrow \quad 1+(2\pi f_1 RC)^2 = 1.108$$

$$\therefore \quad (2\pi f_1 RC)^2 = 0.108$$

$$(f_1 \times 10^{-6} \times 10^3)^2 = \frac{0.108}{4 \times \pi^2}$$

$$\Rightarrow \boxed{f_1 = 52.2\ \text{Hz}}$$

b) Let $t_g(f)$ denote the group delay of RC LPF and $f_2 = 100\,\text{Hz}$, then $t_g(f_2)$ in msec, is ______.

Soln:

$$\angle H(f) = -\tan^{-1}(2\pi f RC). = \theta(f).$$

$$\therefore \quad t_g = -\frac{d\theta(f)}{df}$$

$$= +\frac{1}{1+(2\pi f RC)^2}$$

$$= \frac{1}{1+(2\times\pi\times 100\times 10^{-6}\times 10^3)^2}$$

$$\boxed{t_g = 0.717\ \text{sec}}$$

$\boxed{P.4.3.5}$ The input to a Channel is a band-Pass signal. It is obtained by linearly modulating a sinusoidal Carrier with a single-tone signal. The output of the Channel due to this input is given by

$$y(t) = \frac{1}{100}\cos(100t - 10^{-6})\cos(10^6 t - 1.56).$$ The group delay (t_g) & the phase delay (t_p) in seconds, of the Channel are, —?

Soln:
$$y(t) = \frac{1}{100}\cos\Big(\underbrace{100(t - 10^{-8})}_{\text{msg signal}}\Big).\cos\Big(\underbrace{10^6(t - 1.56 \times 10^{-6})}_{\text{Carrier}}\Big).$$

So, $\boxed{t_g = 10^{-8}}$ $\boxed{t_p = 1.56 \times 10^{-6}.}$

⇒ Message signal gives t_g & Carrier signal gives t_p.

$\boxed{Q}$ Which of the following below is distortion less?

Sol:
(A) $\theta(\omega) = -\omega^2 + \omega^3$.
(B) $\theta(\omega) = \ln\omega$.
(C) $\theta(\omega) = e^{\omega}$.

} Non-linear

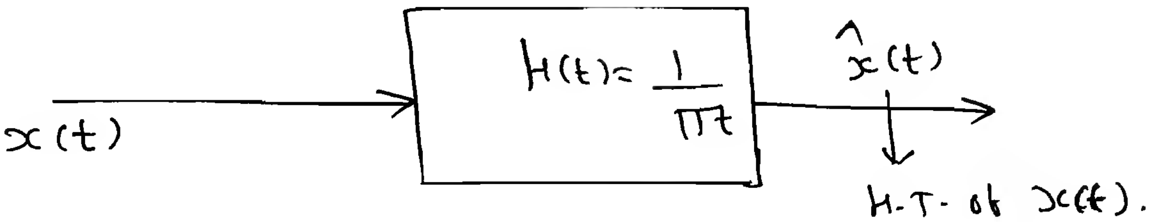
(D) $\theta(\omega) = -3\omega \longrightarrow$ Linear

Soln: Ans - (D) → $\theta(\omega) = \underbrace{-3\omega}_{\text{Linear.}}$

2 Hilbert Transform:-

⇒ The Hilbert transform is an operation that shifts the phase of $x(t)$ by $-\pi/2$, while the amplitude spectrum of the signal remains unaltered.

⇒



⇒ $\hat{x}(t) = x(t) * \frac{1}{\pi t}$.

$\downarrow$ F.T.

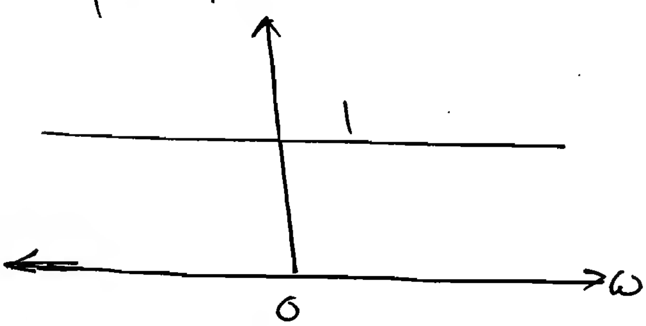
$\hat{X}(\omega) = X(\omega) \cdot \left[-j\,\mathrm{sgn}(\omega)\right]$.

$\therefore$ freq. response of H.T. $= \boxed{H(\omega) = \frac{\hat{X}(\omega)}{X(\omega)} = -j\,\mathrm{sgn}(\omega).}$

⇒ $H(\omega) = -j; \quad \omega > 0$

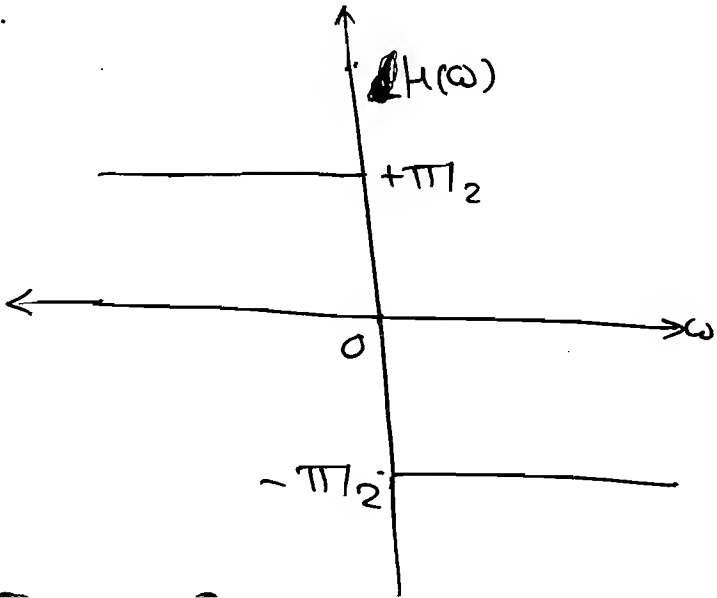
$= +j; \quad \omega < 0$.

$|H(\omega)| = 1$



⇓

All pass filter



⇒ An ideal H.T. is an all pass 90° phase shifter.

⇒ It is obeying orthogonality,

Area under two signal must be zero.

i.e. $$\boxed{\int_{t_1}^{t_2} x(t).\hat{x}(t).dt = 0.}$$

* Properties of H.T.

1) H.T. doesn't change the domain of a signal.

2) H.T. doesn't alter the amplitude spectrum of a signal.

3) If $\hat{x}(t)$ is H.T. of $x(t)$, then H.T. of $\hat{\hat{x}}(t)$ is $-x(t)$.

4) $x(t)$ and $\hat{x}(t)$ are orthogonal to each other.

$\boxed{P4.4.1}$ Find the H.T. of

(1) $x(t) = \cos\omega_0 t$

(2) $x(t) = \sin\omega_0 t$.

Soln:

$$x(t) = \cos\omega_0 t \xleftrightarrow{H.T.} \cos(\omega_0 t - \pi/2)$$
$$= \cos(\pi/2 - \omega_0 t).$$
$$= \sin\omega_0 t.$$

$$x(t) = \sin\omega_0 t \xleftrightarrow{H.T.} \sin(\omega_0 t - \pi/2).$$
$$= -\sin(\pi/2 - \omega_0 t).$$
$$= -\cos\omega_0 t.$$

$\Rightarrow$ H.T. of $e^{j\omega_0 t}$ $(\omega_0 > 0)$ $= -j e^{j\omega_0 t}$

$\downarrow$

$e^{j(\omega_0 t - \pi/2)} = e^{j\omega_0 t} \cdot e^{-j\pi/2}$

$= e^{j\omega_0 t} \cdot (-j).$

$= (-j) \cdot e^{j\omega_0 t}.$

$\Rightarrow$ H.T. of $\delta(t)$ is ______.

$\Rightarrow \delta(t) * \frac{1}{\pi t} = \frac{1}{\pi t}.$

$\Rightarrow$ H.T. of $\frac{1}{\pi t}$ is $\underline{-1}$.

$\longrightarrow \boxed{\text{H.T}} \longrightarrow \boxed{\text{H.T.}} \longrightarrow$

$(-j \,\text{sgn}\, \omega)^2 = j^2 (1)$

$= -1.$

(or)

$= \cos(180^\circ)$

$+ \sin(180^\circ)$

$= -1 + 0$

$= -1.$

* CORRELATION :- (correlogram).

$x(t), x(t-\tau)$ — $\boxed{\text{A.C.F}}$

Auto Correlation f^n

- Energy
- Power

$x(t), y(t-\tau).$ — $\boxed{\text{C.C.F}}$

Cross Correlation f^n.

- Energy
- Power.

⇒ It Provide a measure of the similarity between 2 waveforms as the function of search parameter (τ).

⇒ An application of corelation to signal detection in a radar, when a signal pulse is transmitted in order to detect a suspect target. If a target is present, the pulse will be reflected by it. If the target is not present, there will be no reflection pulse, just noise. By detecting the presence (or) absense of the reflected pulse we confirm the presence (or) absence of target.

⇒

	A.C.F.
Energy	$R_x(\tau) = \int_{-\infty}^{+\infty} x(t) \cdot x(t-\tau)\, dt.$
Power	$R_x(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) \cdot x(t-\tau)\, dt.$

Lag (or) searching parameter

* Properties of ACF :-

1) ACF is an even function of τ

i.e $R_x(\tau) = R_x(-\tau)$.

2) ACF at origin indicates either energy (or) Power in the signal.

3) Max. value of ACF is at origin,

i.e., $|R_x(\tau)| \le |R_x(0)| \;\forall \tau$.

4) $R_x(\tau) = x(\tau) * x(-\tau)$.

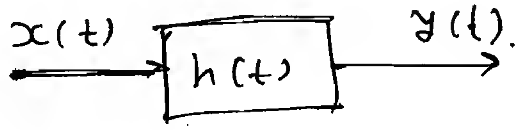
$$x(\tau) * x(-\tau) = \int_{-\infty}^{\infty} x(t) \cdot x(-(\tau - t)) \cdot dt.$$

$$= \int_{-\infty}^{+\infty} x(t) \cdot x(t-\tau) \cdot dt = R_x(\tau).$$

5) F.T. of ACF is known as ESD (or) PSD

$$R_x(\tau) \xleftrightarrow{F.T.} S_x(\omega) \quad \text{ESD} \,|\, \text{PSD}.$$

6) For an LTI system



$\therefore Y(\omega) = X(\omega) \cdot H(\omega)$.

$\therefore |Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2$.

$$\therefore \quad |Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2.$$

$$\therefore \quad S_Y(\omega) = S_X(\omega) \cdot |H(\omega)|^2.$$

Output Spectral density = [input spectral density] $\times$ $[|H(\omega)|^2]$.

$\boxed{P\,4.5.1}$ Find the Auto correlation and power in the signal.

$$x(t) = 6\cos\left(6\pi t + \frac{\pi}{3}\right).$$

Soln:

$$P_{avg} = \frac{A^2}{2} = \frac{36}{2} = 18\ W.$$

$$\rightarrow R_x(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 6\cos\left(6\pi t + \frac{\pi}{3}\right) \cdot 6\cos\left(6\pi t - 6\pi\tau + \frac{\pi}{3}\right) dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{36}{2} \cdot \left[\cancel{\cos\left(12\pi t - 6\pi\tau + \frac{\pi}{3}\right)} \xrightarrow{} 0 + \cos(6\pi\tau)\right] dt.$$

$$= \lim_{T\to\infty} \frac{18}{2T} \int_{-T}^{T} \cos(6\pi\tau) \cdot dt.$$

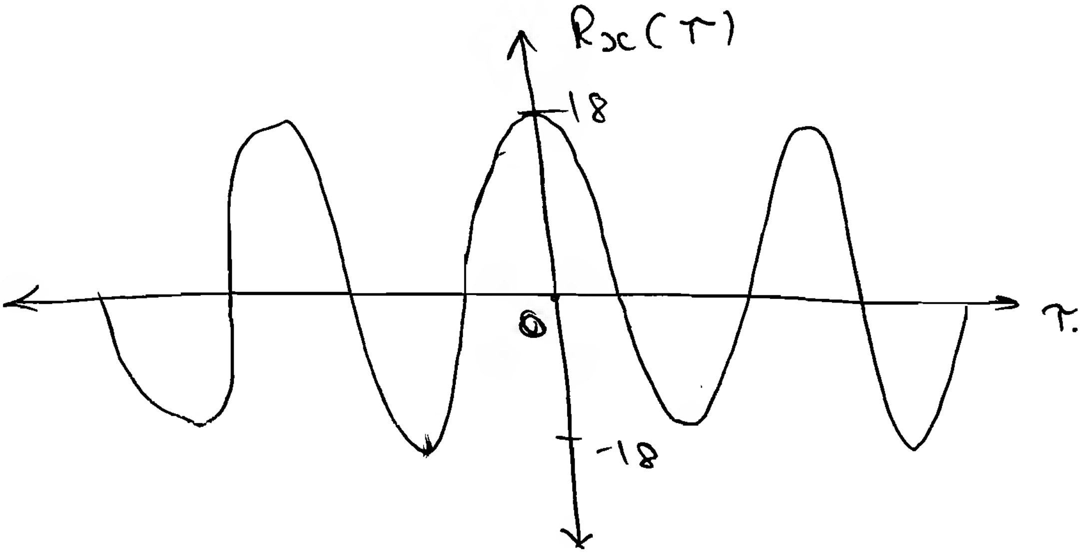
$$= \lim_{T\to\infty} \frac{18\cos(6\pi\tau)}{2T} \times \int_{-T}^{T} dt.$$

$$= \lim_{T\to\infty} \frac{18\cos(6\pi\tau)}{\cancel{2T}} \times \cancel{2T}$$

$\Rightarrow \quad \boxed{R_x(\tau) = 18\cos(6\pi\tau).}$

(or) Power $= R_x(0) = 18$ W.

$$\boxed{\left.\begin{array}{l} A\cos(\omega_0 t+\theta) \\ \text{(or)} \\ A\sin(\omega_0 t+\theta) \end{array}\right\} \longrightarrow \frac{A^2}{2}\cos(\omega_0\tau) \leftarrow \text{ACF is fixed.}}$$



$\boxed{\text{P 4.5.2.}}$ Find the ACF of $x(t) = e^{-3t}u(t)$.

$\downarrow$ energy signal.

Soln:

$$R_x(t) = \int_{-\infty}^{\infty} x(t)\cdot x(t-\tau)\cdot dt.$$

$$= \int_{0}^{\infty} e^{-3t} \cdot e^{-3(t-\tau)} \cdot dt$$

wrong

$$= e^{3\tau}\int_{0}^{\infty} e^{-6t}\cdot dt.$$

$$= \frac{-e^{3\tau}}{6}[0-1] = \frac{e^{3\tau}}{6}.$$

$\Rightarrow \quad x(t) = e^{-3t} \cdot u(t)$ is energy signal.

$$\therefore R_x(\tau) = \int_{-\infty}^{\infty} x(t) \cdot x(t-\tau)\, dt.$$

$$\therefore R_x(\tau) = \int_{-\infty}^{\infty} e^{-3t} \cdot u(t) \cdot e^{-3(t-\tau)} \cdot u(t-\tau) \cdot dt$$

$u(t) \cdot u(t-\tau)$

$t > \tau$

& $\quad t < \tau.$

(OR)

$$\Rightarrow R_x(\tau) = x(\tau) * x(-\tau).$$

$$= e^{-3\tau} \cdot u(t) * e^{3\tau} \cdot u(-\tau).$$

F.T. $\downarrow$

$$= \left[\frac{1}{3+j\omega}\right] \times \left[\frac{1}{3-j\omega}\right].$$

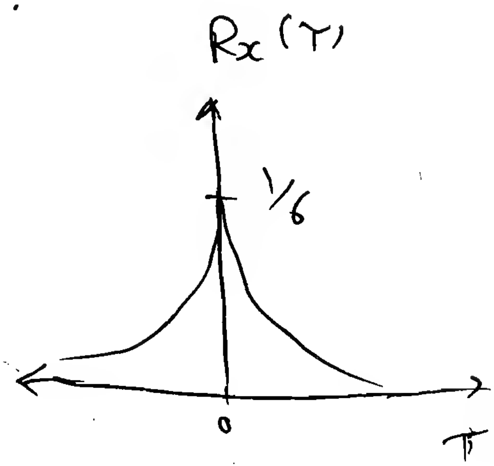
$$= \left(\frac{1}{\omega^2+9}\right)$$

$$= \frac{1}{6}\left(\frac{2(3)}{\omega^2+(3)^2}\right).$$

$\downarrow$ I.F.T

$$\therefore \boxed{R_x(\tau) = \frac{1}{6} e^{-3|\tau|}.}$$

$$\therefore \boxed{R_x(0) = \text{Energy} = 1/6.}$$

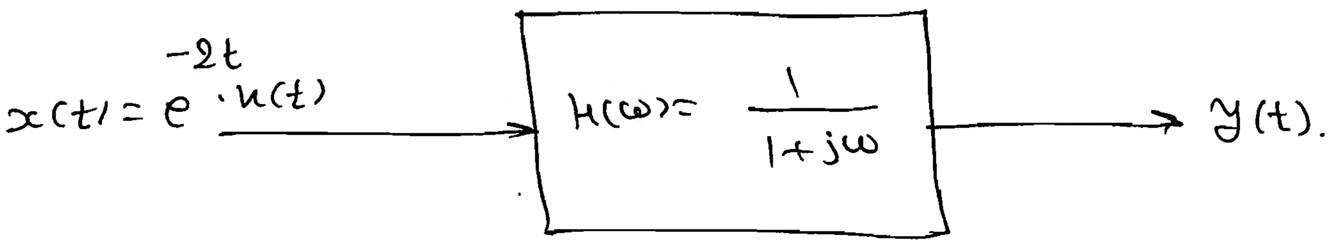


Q P. 4.5.3 Consider a filter with $H(\omega) = \frac{1}{1+j\omega}$ and input $x(t) = e^{-2t} \cdot u(t)$.

(a) Find the ESD of the output?

(b) Show that total energy in the o/p is one-third of the input energy?

Soln:



$\therefore x(t) = e^{-2t} \cdot u(t)$

$\downarrow$ F.T.

$\therefore X(\omega) = \frac{1}{2+j\omega}$.

$Y(\omega) = X(\omega) \cdot H(\omega)$.

$|Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2$.

$$\therefore S_y(\omega) = |H(\omega)|^2 \cdot S_x(\omega).$$

$$S_x(\omega) = |X(\omega)|^2 = \frac{1}{4+\omega^2}.$$

$$\therefore |H(\omega)|^2 = \frac{1}{1+\omega^2}.$$

$$\therefore \boxed{S_y(\omega) = \left(\frac{1}{1+\omega^2}\right) \times \left(\frac{1}{4+\omega^2}\right).}$$ ESD at o/p.

(b) Energy = Area under ESD.

$$\therefore E_y = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_y(\omega) \cdot d\omega.$$

$$\therefore \; E_y(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{(\omega^2+1)} \cdot \frac{1}{(\omega^2+4)} \cdot d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\frac{1}{3}}{\omega^2+1} - \frac{\frac{1}{3}}{\omega^2+4} \cdot d\omega.$$

$$= \frac{1}{2\pi} \times \frac{1}{3}\left[\tan^{-1}(\omega) - \frac{1}{2}\tan^{-1}(\omega/2)\right]_{-\infty}^{\infty}$$

$$= \frac{1}{6\pi}\left[\frac{\pi}{2} - \frac{\pi}{4} - \frac{\pi}{2} - \frac{\pi}{4}\right].$$

$$= \frac{1}{6\pi}\left[-\frac{\pi}{2}\right].$$

$$\therefore \; \boxed{E_y(\omega) = \frac{1}{12}.}$$

$\Rightarrow$ I/P $\quad x(t) = e^{-2t}\,u(t).$

$$E_x = \int_0^{\infty} (e^{-2t})^2 \cdot dt$$

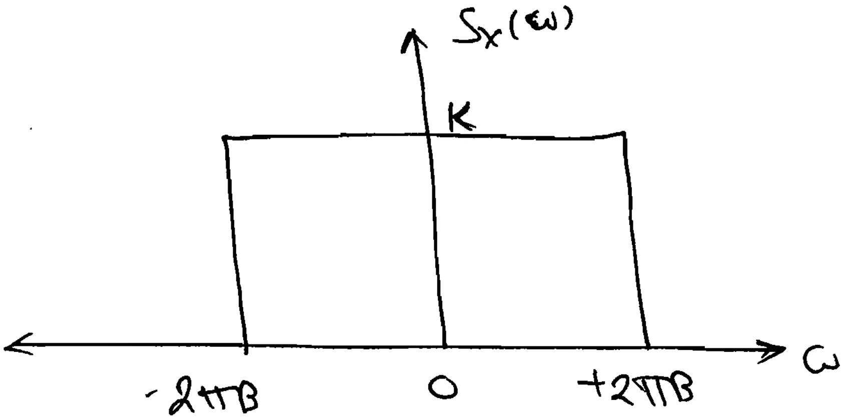
$$= \int_0^{\infty} e^{-4t} \cdot dt$$

$$\boxed{E_x = 1/4}$$

So, $\boxed{E_{O/P} = \frac{1}{3}\, E_{I/P}}$

P 4.5.4 A power signal whose psd is shown in fig. is applied to an ideal

differentiator, find the mean square value of the o/p of the differentiator.



Soln:

$$x(t) \longrightarrow \boxed{\frac{d}{dt} \xrightarrow{F.T} j\omega} \longrightarrow y(t).$$

$\Rightarrow$ mean square value of o/p

$$= \text{Power} = \text{Area under o/p PSD}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_y(\omega).\, d\omega.$$

Now, $S_y(\omega) = |H(\omega)|^2 . S_x(\omega).$

so, $|H(\omega)|^2 = \omega^2.$

$$\therefore \text{M.S.V. of o/p} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \omega^2 . S_x(\omega).\, d\omega.$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \omega^2 . S_x(\omega) . d\omega.$$

$$= \frac{1}{2\pi} \int_{-2\pi B}^{2\pi B} \omega^2 . k\, d\omega.$$

$$= \frac{1}{2\pi} \times 2 \times k \times \int_0^{2\pi B} \omega^2 . d\omega$$

$$= \frac{k}{\pi} \times \left[\frac{\omega^3}{3} \right]_0^{2\pi B}$$

$$= \frac{k}{\pi} \times \frac{8 \times \pi^3 \times B^3}{3}$$

$$\boxed{\text{M.S.V. of o/p} = \frac{8\pi^2 B^3 K}{3}}$$

$\boxed{P\,4.5.5}$ Find the C.C.F of $x(t) = e^{-t} u(t)$ and $y(t) = e^{-3t} u(t)$?

Soln:

ACF $\Rightarrow R_x(\tau) = x(\tau) * x(-\tau)$.

CCF $\Rightarrow R_{xy}(\tau) = x(\tau) * y(-\tau)$.

&

$R_{yx}(\tau) = Y(\tau) * x(-\tau)$.

$\therefore \boxed{R_{yx} = R_{xy}}$

So, $R_{xy}(\tau) = x(\tau) * y(-\tau)$.

$$\downarrow \text{F.T.} \quad = \left[e^{-t} u(t) \right] * \left[e^{3t} u(-t) \right].$$

$$= \frac{1}{1 + j\omega} \times \frac{1}{3 - j\omega}.$$

$$= \frac{1}{(1 + j\omega)(3 - j\omega)}.$$

$$= \frac{1}{4}\left[\frac{1}{1+j\omega} + \frac{1}{3-j\omega}\right].$$

$\downarrow$ I.F.T.

$$\boxed{R_x(t) = \frac{1}{4}\left[e^{-t}\cdot u(t) + e^{3t}\cdot u(-t)\right].}$$

* F.T. of Periodic Signals :-

$\Rightarrow$ Periodic and discrete in one domain is corresponds to discrete & periodic in other domain.

$$\Rightarrow \quad 1 \xrightarrow{F.T.} 2\pi\,\delta(\omega).$$

$$1.\, e^{j\omega_0 t} \xrightarrow{F.T.} 2\pi\,\delta(\omega-\omega_0)$$

$$\Rightarrow \quad \cos\omega_0 t = \frac{1.\,e^{j\omega_0 t} + 1.\,e^{-j\omega_0 t}}{2}$$

$$= \frac{1}{2}.\, e^{j(1)(\omega_0)t} + \frac{1}{2}.\, e^{j(-1)(\omega_0)t}.$$

$\downarrow$ F.T.

$$= \frac{2\pi}{2}\,\delta(\omega-\omega_0) + \frac{2\pi}{2}.\,\delta(\omega+\omega_0).$$

$$\cos\omega_0 t \xrightarrow{F.T.} = 2\pi\left[\frac{\delta(\omega-\omega_0) + \delta(\omega+\omega_0)}{2}\right].$$

Rect	$\xleftrightarrow{F.T.}$	Sa (or) Sinc
Δ^{le}	$\xleftrightarrow{F.T.}$	Sa^2 (or) $Sinc^2$.

Impulse $\xleftrightarrow{F.T.}$ Constant

Imupulse train $\xleftrightarrow{F.T.}$ Impulse train

Gaussian $\xleftrightarrow{F.T.}$ Gaussian.

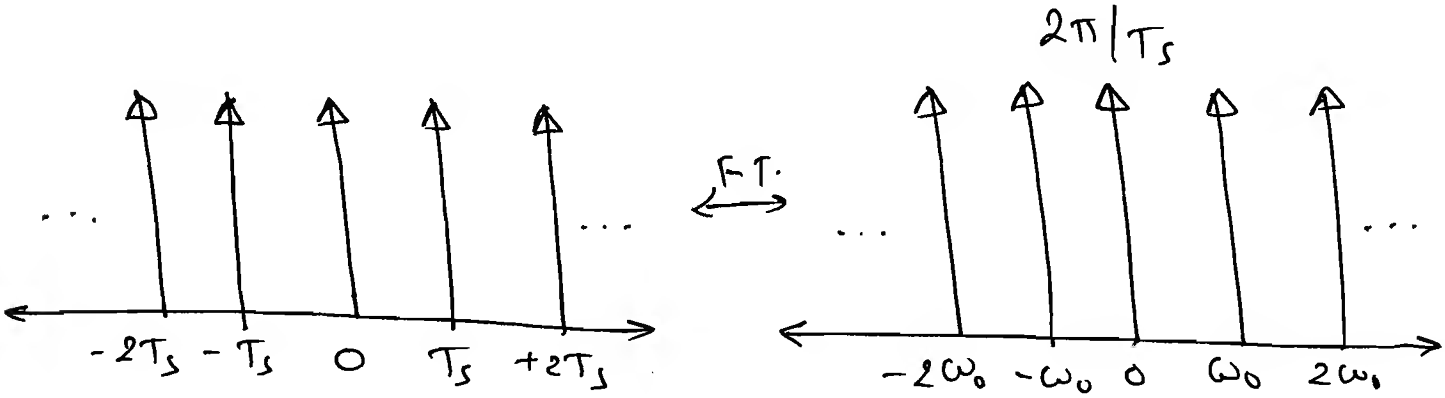
$\Rightarrow$

$$x_p(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t}.$$

$\downarrow$ F.T.

$$X_p(\omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n \cdot \delta(\omega - n\omega_0).$$

$\Rightarrow$ F.T. of a Periodic signal consist of a seauence of eauidistant impulse located at harmonic frequencies of the signal.



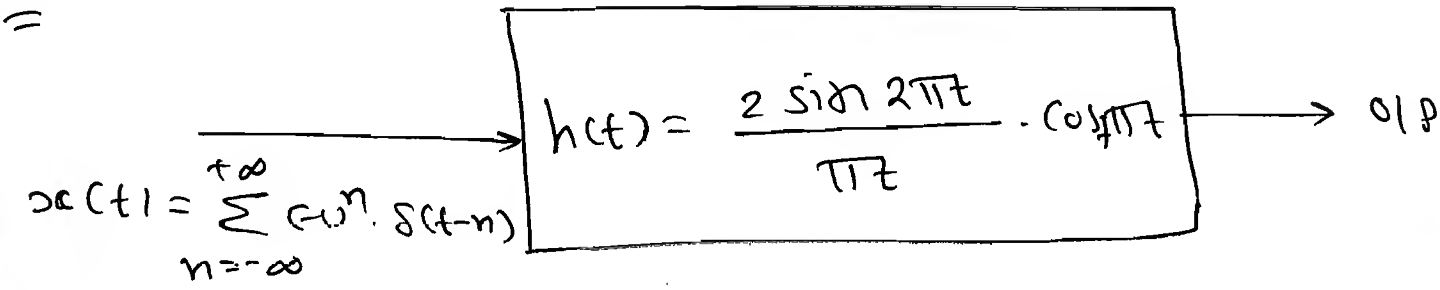
$\Rightarrow$

$$\sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \longleftrightarrow \frac{2\pi}{T_s} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_0).$$

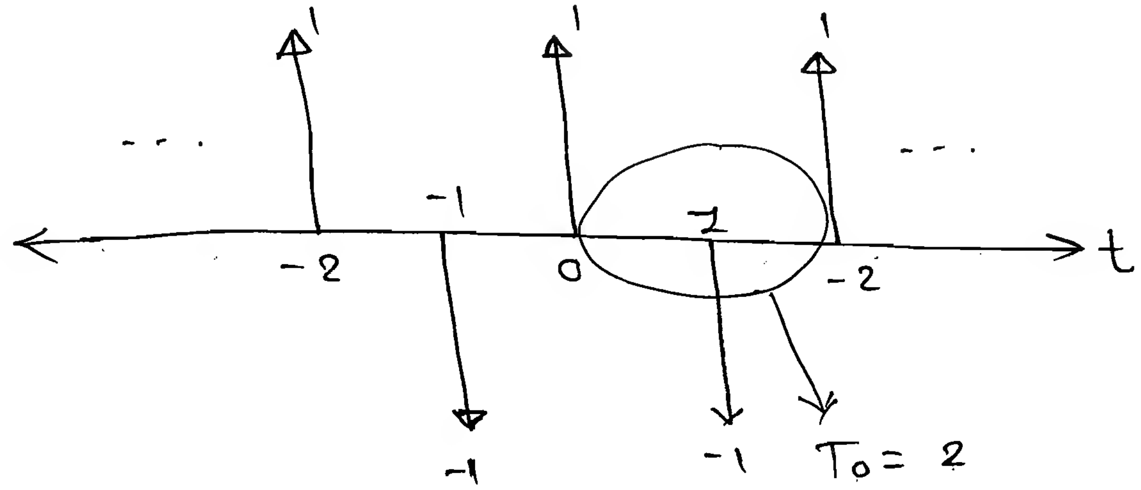
$\boxed{a}$ An L.T.I. sys. is having Impulse Response $h(t) = 2\frac{\sin 2\pi t}{\pi t} \cdot \cos 7\pi t$ for which the I/P applied is

$$x(t) = \sum_{n=-\infty}^{+\infty} (-1)^n \delta(t-n),$$ find the O/P.

Soln:



$$\Rightarrow x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-n).$$



$T_0 = 2$

$\omega_0 = \frac{2\pi}{T} \Rightarrow \boxed{\omega_0 = \pi}$

$\Rightarrow$ Now,

$$X_p(\omega) = 2\pi \sum_{n=-\infty}^{+\infty} C_n \cdot \delta(\omega - n\omega_0).$$

So, $$C_n = \frac{1}{T}\int_0^T x(t) \cdot e^{-j\omega_0 n t}.$$

$$\Rightarrow C_n = \frac{1}{2}\int_0^2 [\delta(t) - \delta(t-1)]\, e^{-jn\pi t}\, dt.$$

$$= \frac{1}{2}\left[\int_0^2 \underbrace{\delta(t)\, e^{-jn\pi t}}_{t_0=0}\, dt - \int_0^2 \underbrace{\delta(t-1)\, e^{-jn\pi t}}_{t_0=1}\, dt\right]$$

shifting property of impulse.

$$\Rightarrow C_n = \frac{1}{2}\left[e^{-jn\pi(0)} - e^{-jn\pi(1)}\right].$$

$$= \frac{1}{2}\left[1 - e^{-jn\pi}\right].$$

$$C_n = \frac{1}{2}\left[1 - (-1)^n\right]$$

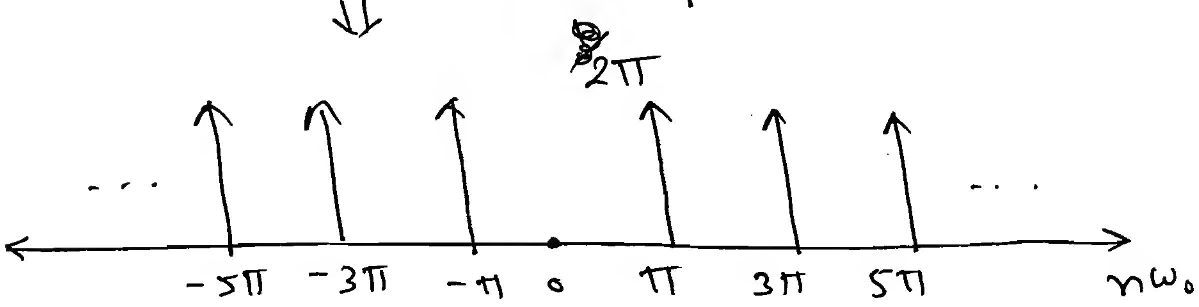
$$\Rightarrow C_n = 1 \quad ; \quad n = \text{odd}.$$

$$= 0 \quad ; \quad n = \text{even}.$$

$$\therefore X_p(\omega) = 2\pi \sum_{n=-\infty}^{+\infty} (1).\ \delta(\omega - n\pi) \qquad (\because \omega_0 = \pi).$$

(odd n) $\Downarrow$ & $T_0 = 1$

$X_p(\omega)$.



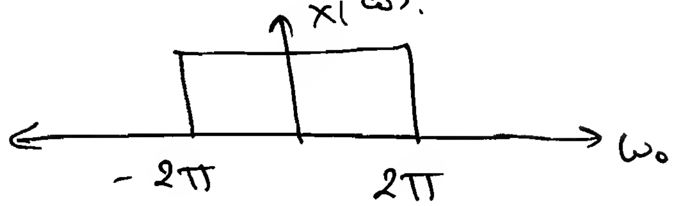
Now, $h(t) = 2\,\dfrac{\sin 2\pi t}{\pi t} \cdot \cos 7\pi t.$

$$= 2\,\frac{\sin 2\pi t}{\pi t} \cdot \left[\frac{e^{-j7\pi t} + e^{j7\pi t}}{2}\right]$$

$$h(t) = \frac{\sin 2\pi t}{\pi t} \cdot \left[\frac{e^{-j\pi 7} + e^{j\pi 7}}{1} \right]$$

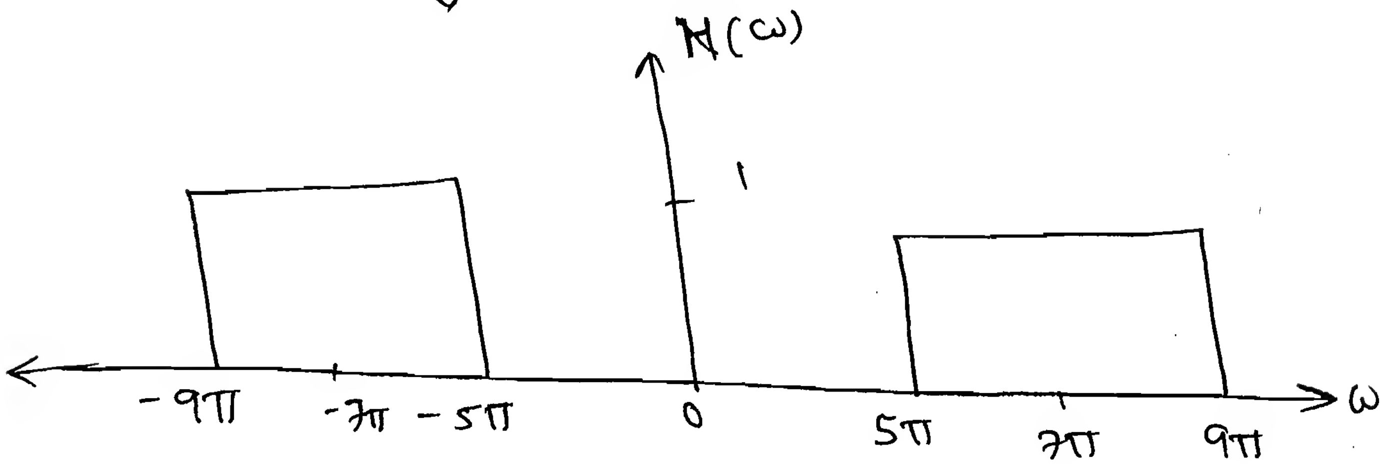
ie, $x(t) = \dfrac{\sin 2\pi t}{\pi t}$

$\downarrow$ F.T.

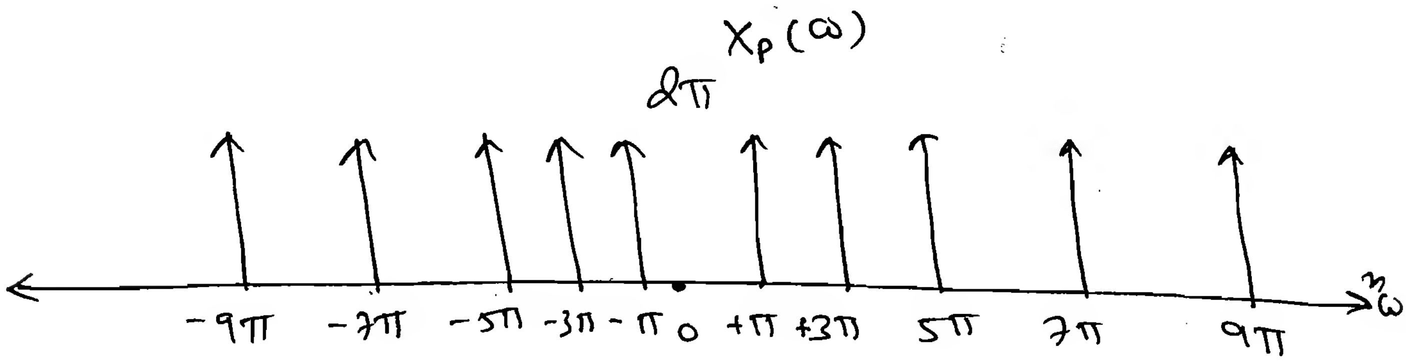


$$\therefore \quad H(\omega) = X(\omega - 7\pi) + X(\omega + 7\pi).$$

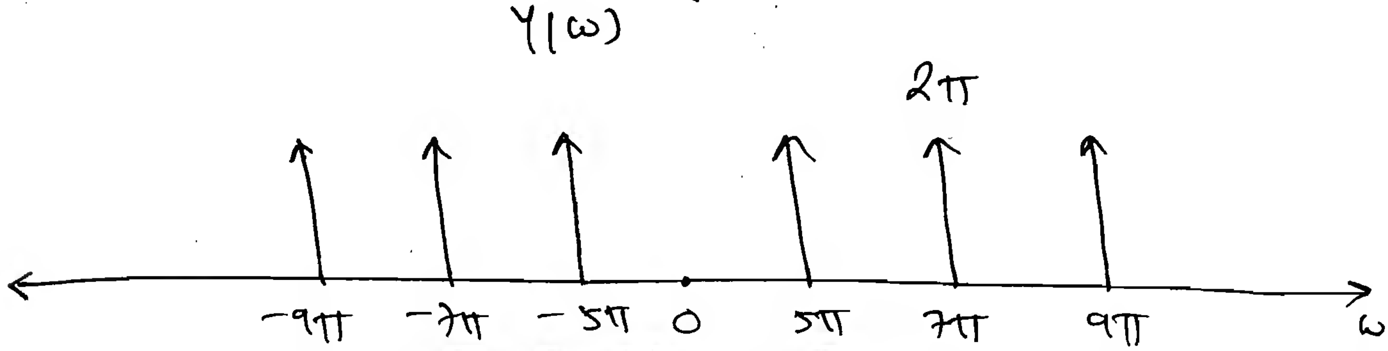
$\Downarrow$



$\times$



$\Downarrow$



$$\Rightarrow \quad Y(\omega) = 2\pi \big[\delta(\omega - 5\pi) + \delta(\omega + 5\pi) + \delta(\omega - 7\pi) + \delta(\omega + 7\pi) + \delta(\omega - 9\pi) + \delta(\omega + 9\pi) \big].$$

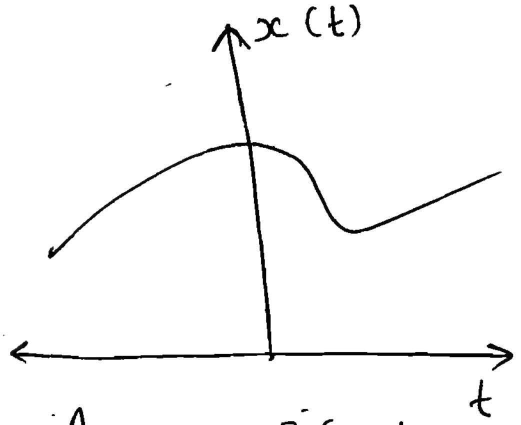
$$\therefore \boxed{y(\omega) = 2\left[\cos 5\pi t + \cos 7\pi t + \cos 9\pi t\right].}$$

* Sampling Theorem :-

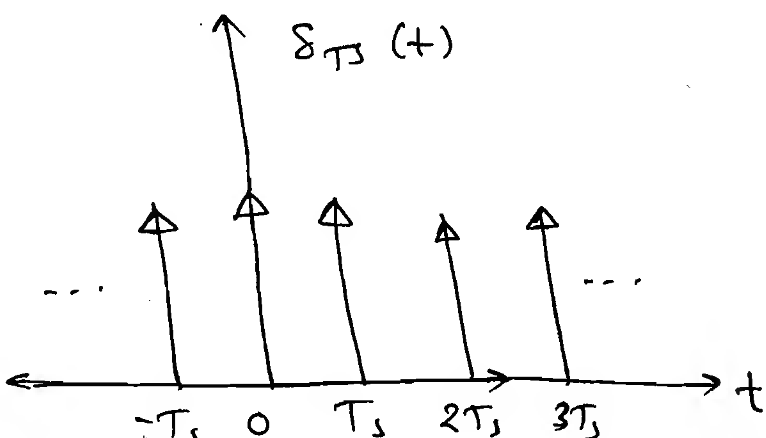
- Ideal
- Practical — $\omega_s > 2\omega_m$ (or) $f_s > 2 f_m$
  - Natural
  - Flat-top.

① Ideal Sampling :-

$\Rightarrow$

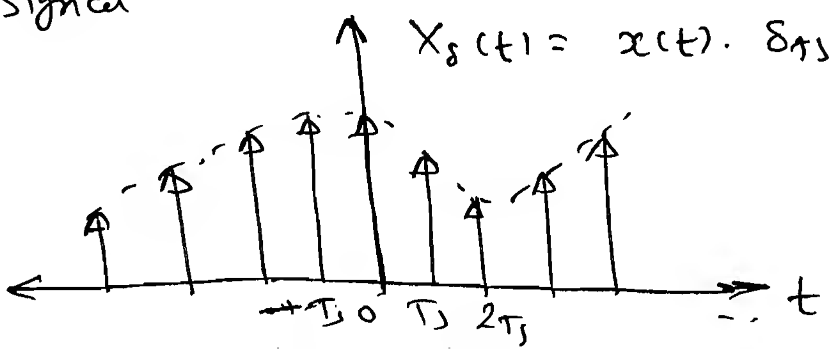


Analog signal



Sampling fn.

$$X_s(t) = x(t) \cdot \delta_{T_s}(t).$$



Ideal Sampled signal

$\Rightarrow$ $x(t) \rightarrow$ Analog signal

$\Rightarrow$ $$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$\delta_{T_s}(t) =$ Samping function.

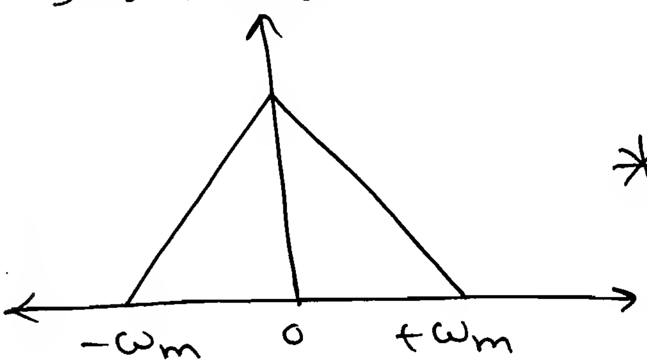
$\Rightarrow$ $x_\delta(t) \rightarrow$ Ideally Sampled signal.

$$\boxed{x_\delta(t) = x(t) \cdot \sum_{n=-\infty}^{+\infty} \delta(t - nT_s).}$$ $\leftarrow$ Time domain.

$\Rightarrow$ Take F.T.

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \longleftrightarrow \frac{2\pi}{T_s} \sum_{n=\infty} \delta(\omega - \omega_0 n)$$

$x(t) \leftrightarrow X(\omega)$



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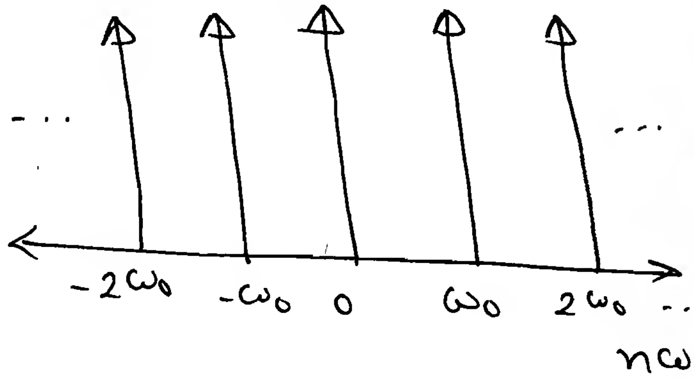
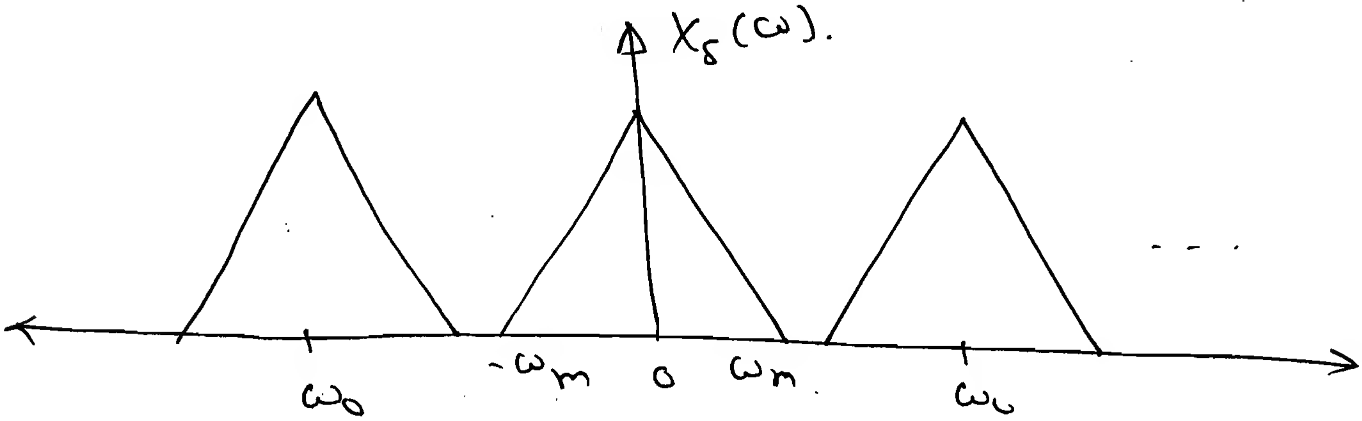


$\Downarrow$ Convolve.



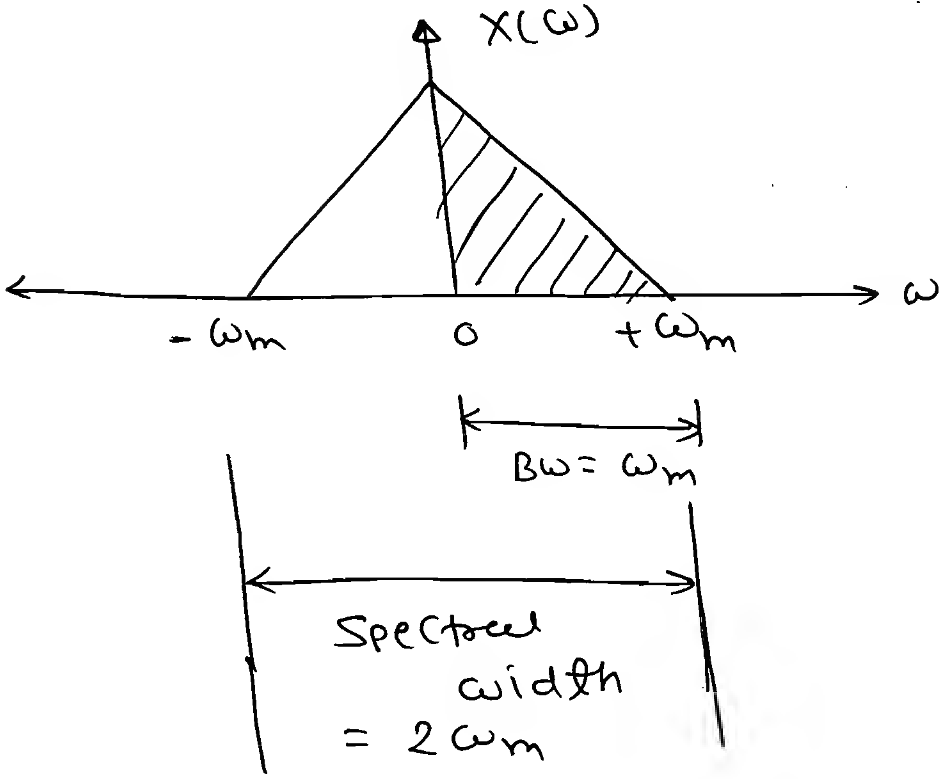
$\Rightarrow$ $$X_\delta(\omega) = \frac{1}{\cancel{2\pi}} \left[X(\omega) * \frac{\cancel{2\pi}}{T_s} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) \right]$$

⇒ Spectrum of $x_\delta(t)$

⇒ $$\boxed{X_\delta(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{+\infty} X(\omega - n\omega_0).}$$

Freq. domain.

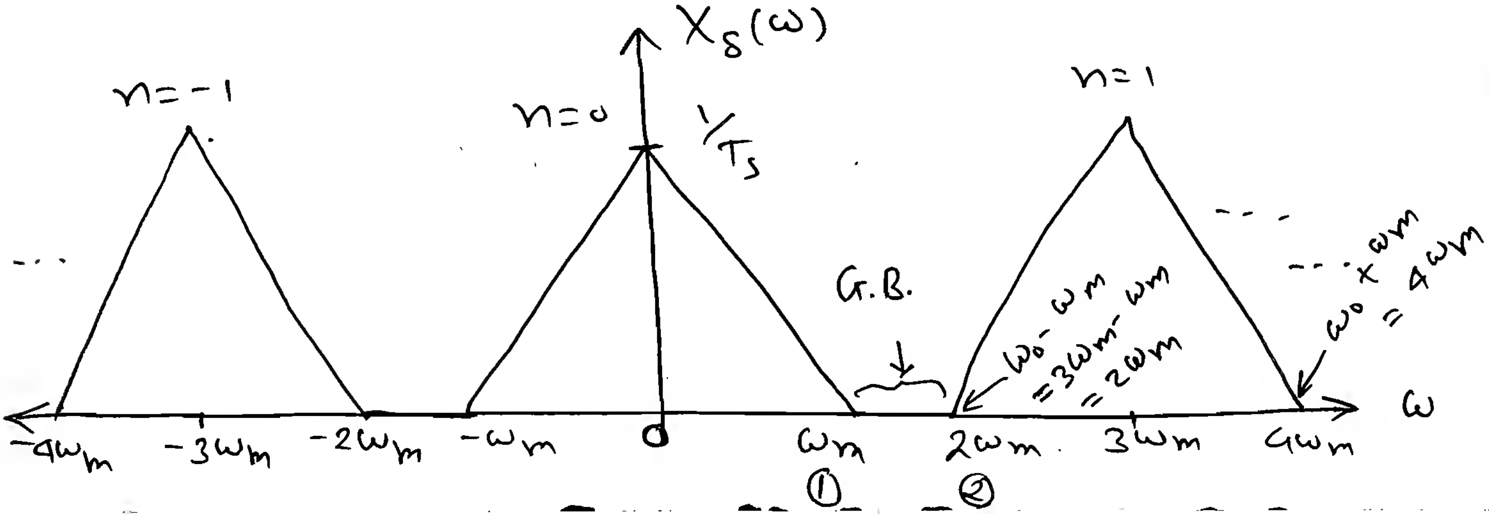
⇒ Assume Band limited spectrum,



<u>case-(i):</u> <u>$\omega_0 > 2\omega_m$.</u> ⇒ <u>over</u> <u>Samping</u>

⇒ Let, $\omega_0 = 3\omega_m$

$$X_\delta(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - 3n\omega_m).$$



$\Rightarrow$ Guard Band = ② − ①

$= (\omega_0 - \omega_m) - (\omega_m)$

$$\boxed{\text{Guard Band} = \omega_0 - 2\omega_m.}$$

$\Rightarrow$ Images don't overlap,

if ~~ω_0~~ ② > ①.

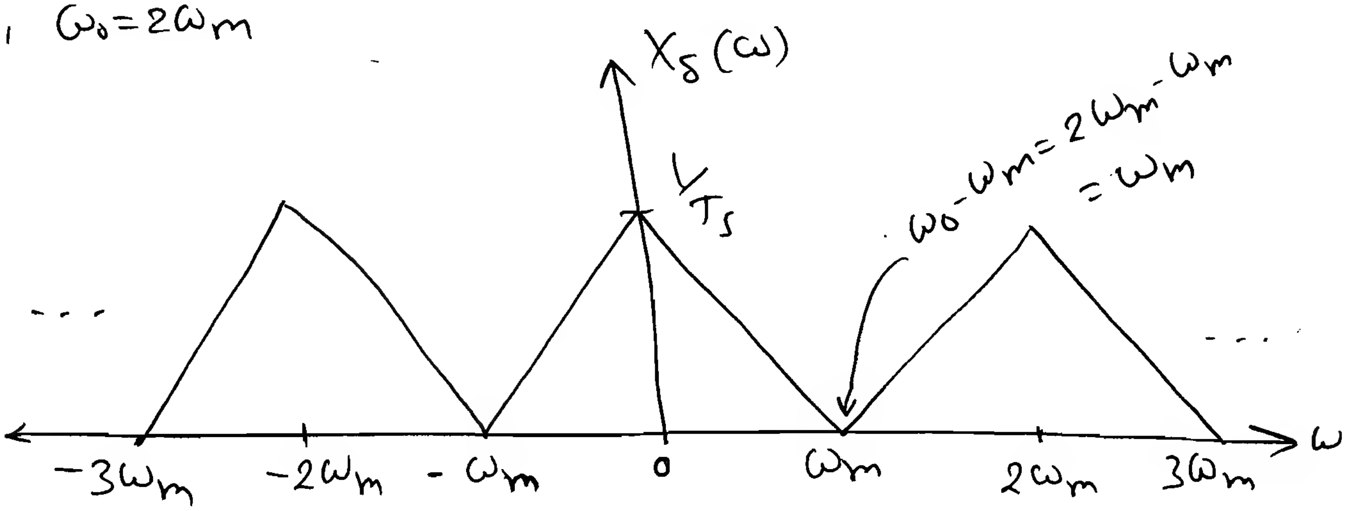
i.e. $\omega_0 - \omega_m > \omega_m.$

$\Rightarrow$ $\boxed{\omega_0 > 2\omega_m.}$

Case-(ii):- $\omega_0 = 2\omega_m$, Critical Sampling.

$$\Rightarrow X_\delta(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X(\omega - 2\omega_m n).$$

let, $\omega_0 = 2\omega_m$



$\Rightarrow$ G.B. = $\omega_0 - 2\omega_m = 2\omega_m - 2\omega_m = 0.$

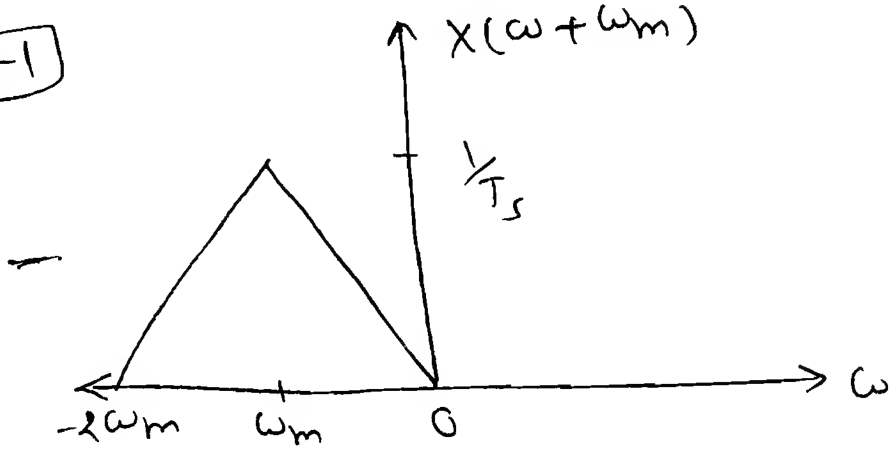
$$\boxed{G.B. = 0}$$

$\Rightarrow$ Case - (iii) : $\omega_0 < 2\omega_m$ Under Sampling.

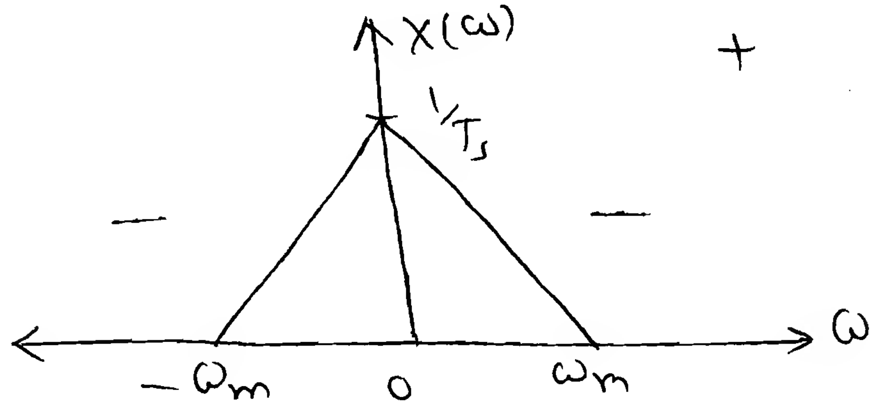
$\Rightarrow$ Let, $\omega_0 = \omega_m$.

$$\therefore \quad X_\delta(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_m).$$

$\boxed{n=-1}$

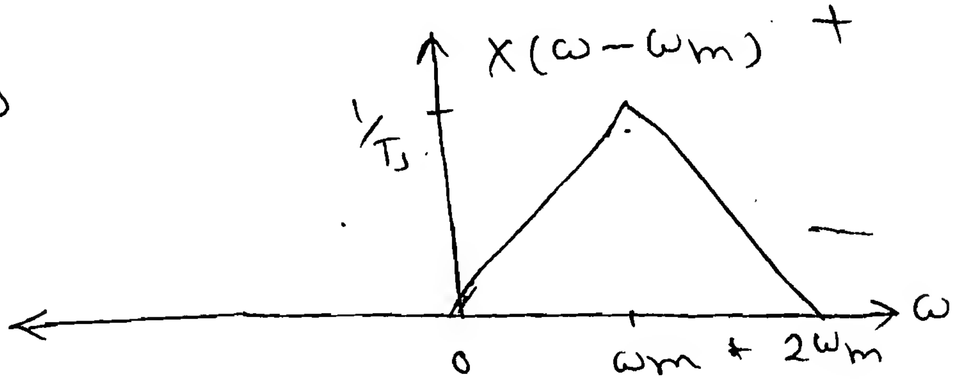


$\boxed{n=0}$



$+$

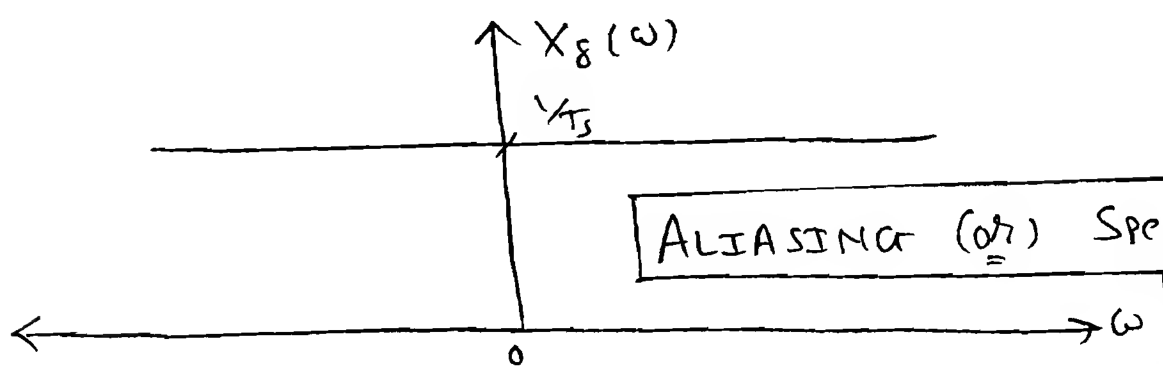
$\boxed{n=1}$



$+$

$\Downarrow$

$=$

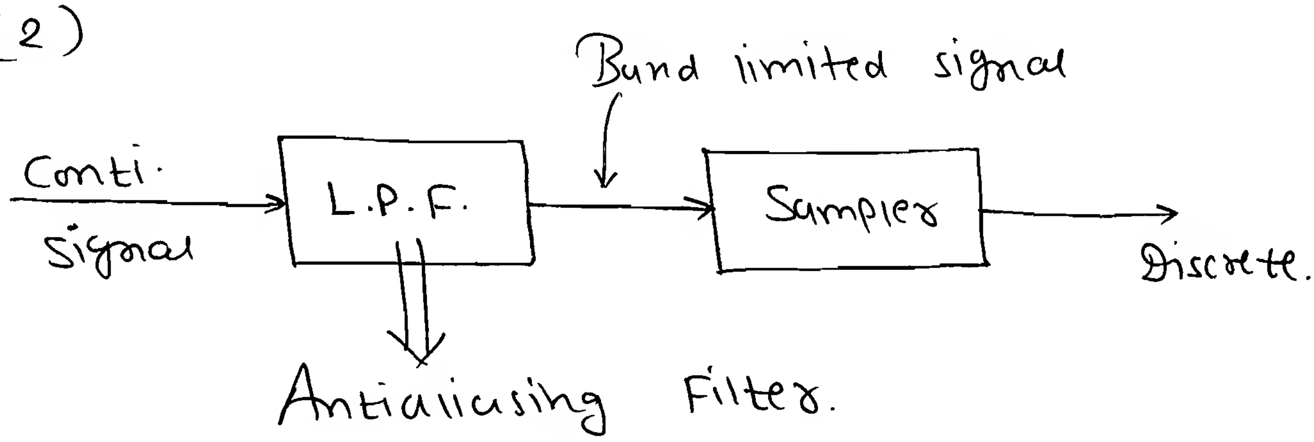


ALIASING (or) Spectral Folding.

⇒ To Avoid Aliasing

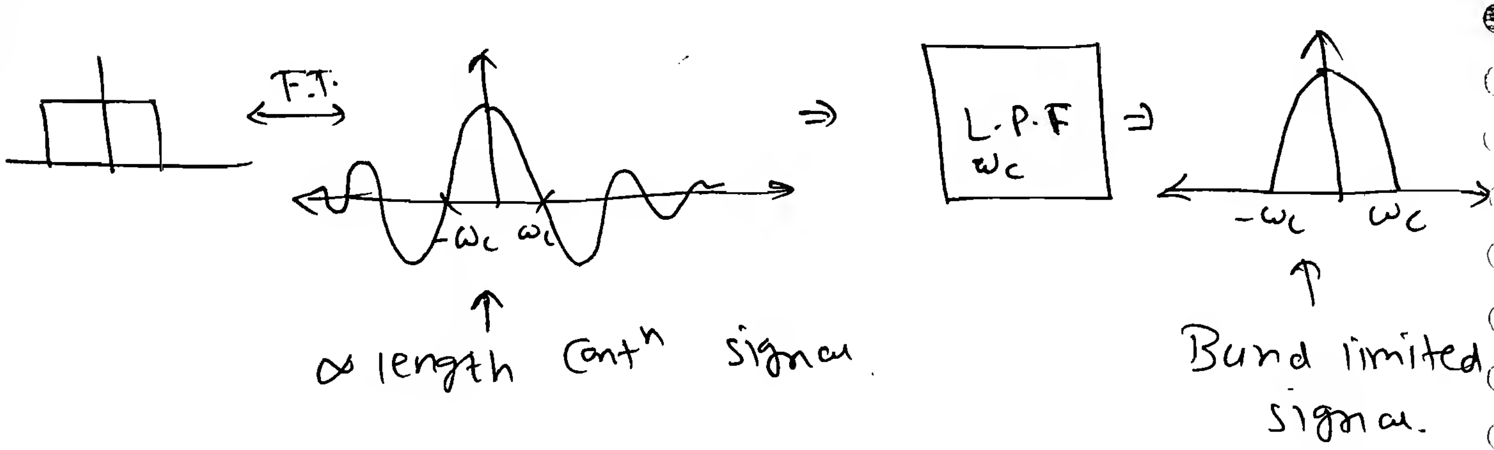
(1) $\boxed{\omega_0 > 2\omega_m.}$

(2)



⇒ No Spectrum in Real time is Band-limited. So, to make ∞ length Spectrum as Band limited, Continuous signal is applied to LPF before Samplinging.

e.g.



⇒ $\boxed{\begin{aligned} \text{Nyquist Rate} &= \text{minimum Sampling Rate} \\ &= 2\,\omega_m \\ &= 2\,(\text{Height freq. of signal component}) \end{aligned}}$

$\boxed{P4.6\text{-}1}$ Find the Nyquist rate & Nyquist interval for each of the following signals:

(a) $x_1(t) = \left(\frac{\sin 200\pi t}{\pi t}\right)$.

Soln: Here, $\omega_m = 200\pi$.

$\therefore$ $N.R = 2\omega_m = 2(200\pi)$. rad/sec.

$\Rightarrow$ $N.R = 200\ Hz. = f_s$

$T_s = \frac{1}{f_s} = \frac{1}{200}$ sec.

$\therefore$ $\boxed{T_s = 5ms}$

(b) $x_2(t) = \left(\frac{\sin 200\pi t}{\pi t}\right)^2$.

Soln: $x_2(t) = \frac{1 + \cos 400\pi t}{4(\pi t)^2}$

So, $\omega_m = 400\pi$

N.R. $\Rightarrow$ $\omega_0 = 2\omega_m = 2(400\pi)$ rad/sec.

$f_s = 2f_m = 400\ Hz.$

$\Rightarrow$ $T_s = \frac{1}{f_s} = \frac{1}{400}$ $\Rightarrow$ $\boxed{T_s = 2.5ms}$

(c) $x_3(t) = 5\cos 1000\pi t \cdot \cos 4000\pi t$.

Soln: $x_3(t) = \frac{5}{2} \cdot 2\cos 1000\pi t \cdot 4000\pi t$.

$= \frac{5}{2} \cdot (\cos 4000\pi t + \cos 5000\pi t)$

$\Rightarrow \quad \omega_m = 5000\pi$

$\Rightarrow \quad N.R. = \omega_0 = 2\omega_m = 2(5000\pi)$ rad/sec.

$f_s = 2 f_m = 5\,KHz.$

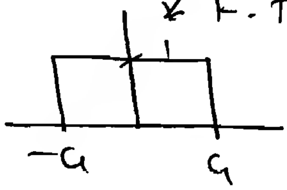
$\Rightarrow \quad T_s = \dfrac{1}{5K} = 0.2\,msec.$

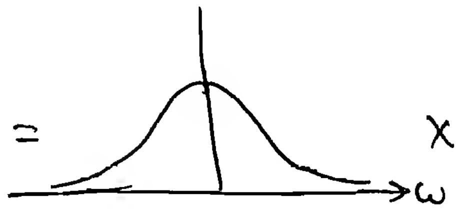
$\boxed{T_s = 0.2\,ms}$

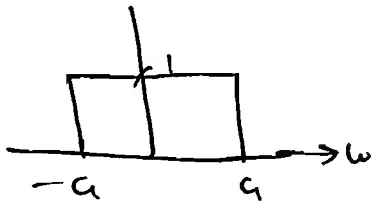
(d) $x_4(t) = e^{-6t}u(t) * \dfrac{\sin at}{\pi t}.$

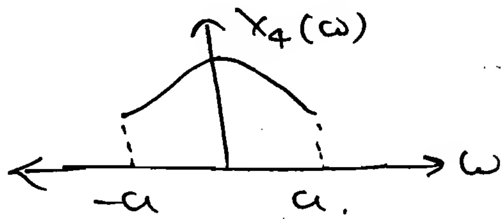
$\downarrow$ F.T $\qquad \downarrow$ F.T

Soln:

$X_4(\omega) = \dfrac{1}{6+j\omega} \times$ 

$=$ 

$\times$ 

$=$ 

$\Rightarrow \quad \omega_m = a.$

$N.R. \Rightarrow \omega_0 = 2\omega_m = 2a.$ rad/sec.

$f_s = 2 f_m = 2 \times \dfrac{\omega_m}{2\pi} = \dfrac{a}{\pi}$ Hz.

$\Rightarrow \quad \boxed{T_s = \dfrac{\pi}{a}\ sec}$

(e) $x_5(t) = \text{sinc}(100t) + 3\,\text{sinc}^2(60t).$

Soln:

$$x_s(t) = \frac{\sin(100\pi t)}{100\pi t} + 3\left(\frac{\sin 60\pi t}{60\pi t}\right)^2.$$

$$x_s(t) = \frac{\sin(100\pi t)}{100\pi t} + 3\left[\frac{1 - \cos 120\pi t}{(2)^2 \times (60\pi t)^2}\right].$$

$\Rightarrow \quad \omega_m = 120\pi$

N.R. $\Rightarrow \quad \omega_0 = 2\omega_m = 2(120\pi)$ rad/sec.

$f_s = 2 f_m = 120$ Hz.

$\Rightarrow \quad \boxed{T_s = \frac{1}{120} \text{ sec.}}$

P 4.6.2. Let $x(t)$ be a signal with Nyquist rate ω_0. Determine the Nyquist rate for each of the following signals.

(a) $x(t) + x(t-1)$.

Soln:

$x(t) \longrightarrow$ B.W. $= \omega_m$.

& N.R $= \omega_0$.

i.e. $\boxed{\omega_0 = 2\omega_m}$

F.T.

$X(\omega) + e^{-j\omega(1)} \cdot X(\omega) \longrightarrow$ N.R. $= \omega_0$

Bw not change hence N.R. not change. i.e. ω_0.

(b) $\frac{d}{dt} x(t)$.

$\Rightarrow$ Soln: $\downarrow$ F.T. $= j\omega X(\omega)$.

$\Rightarrow$ B.w. not change hence N.R. Same.

i.e. ω_0.

(c) $x(3t)$.

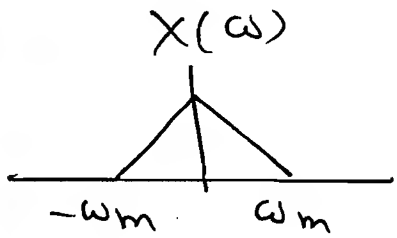
Sol^n:

$$x(3t) \xleftrightarrow{F.T.} \frac{1}{|3|} \cdot X(\omega/3).$$

New, Bw $\Rightarrow$ $\omega_m' = 3\omega_m$.

So New N.R.

$$\omega_0' = 2(3\omega_m)$$
$$= 3(2\omega_m)$$
$$\boxed{\omega_0' = 3\omega_0}$$



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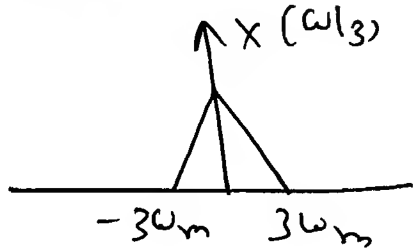
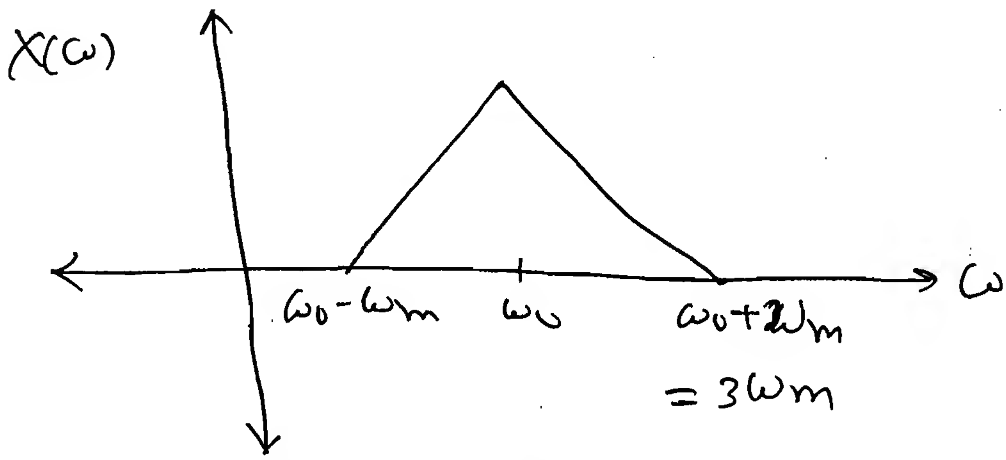


(d) $x(t) \cdot \cos(\omega_0 t)$.

Sol^n:

$$x(t) \cdot \cos(\omega_0 t) \xleftrightarrow{F.T.} \pi\left[X(\omega_m - \overset{2\omega_m}{\cancel{\omega_0}}) + X(\omega_m + \overset{2\omega_m}{\cancel{\omega_0}})\right].$$

eg. $\omega_0 = 2\omega_m$



So, New N.R. $\omega_0' = 2(3\omega_m)$

$= 3(2\omega_m)$

$$\boxed{\omega_0' = 3\omega_0}$$

P4.6.3 Two signals $x_1(t)$ & $x_2(t)$ are band limited to 2 KHz & 3 KHz respectively, find the Nyquist rate of the following signals.

Soln: (a) $x_1(2t)$.

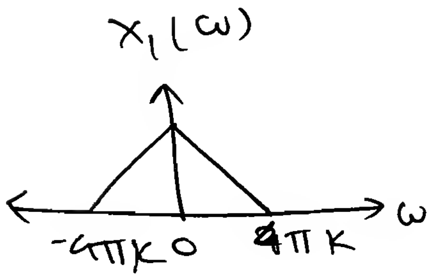
Soln: $x_1(2t) \xleftrightarrow{F.T.} \frac{1}{2} X_1(\omega/2)$.

$x_1(t) \longrightarrow$ B.W. $= 2$ KHz

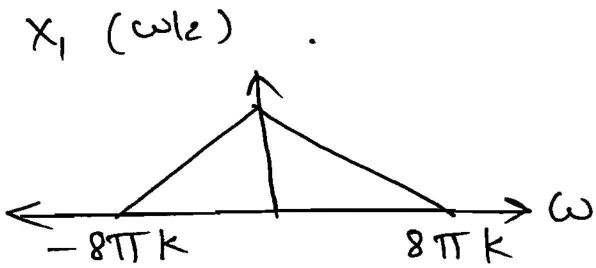
So, N.R. of $x_1(t) \Rightarrow$

$f_{s_1} = 2 f_{m_1} = 4$ KHz.

$\omega_{0_1} = 2\omega_{m_1} = 8\pi$ K rad/sec.



$\longleftrightarrow$



So, N.R. of $x_1(2t)$

$\Rightarrow \omega_{0_1}' = 2\omega_{m_1}' = 2(8\pi)K$

$= 16K\pi$ rad/sec.

$f_{s_1}' = 2 f_{m_1}'$

$$\boxed{f_{s_1}' = 8 \text{ KHz}}$$

(b) $x_2(t-3)$.

Sol^n: B.W. of $x_2(t)$ is $\rightarrow$ 3 KHz.

$\Rightarrow f_{m_2} = 3$ KHz.

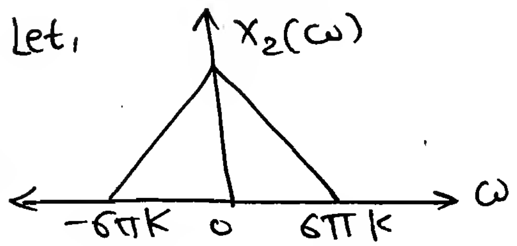
N.R. $f_{s_2} = 2 f_{m_2} = 6$ KHz

$\omega_{o2} = 2\omega_{m_2} = 6(2\pi) = 12\pi k$ rad/sec.

Now, $x_2(t-3) \xleftrightarrow{F.T.} e^{-j3\omega} \cdot X_2(\omega)$.

B.W. is not change. Hence. N.R. is not change.

Let,



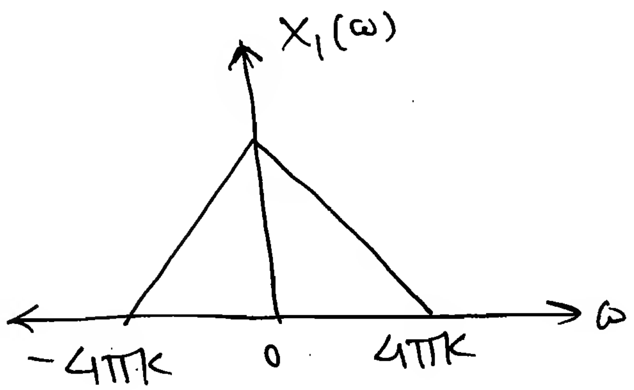
i.e. N.R. of $x_2(t-3)$

$\Rightarrow \omega'_{o2} = 2\omega_{m_2} = 12\pi k$ rad/sec.

& $\boxed{f'_{s_2} = 2 f_m = 6 \text{ KHz}}$

(c) $X_1(t) + X_2(t)$.

Sol^n: $X_1(t) + X_2(t) \xleftrightarrow{F.T.} X_1(\omega) + X_2(\omega)$.



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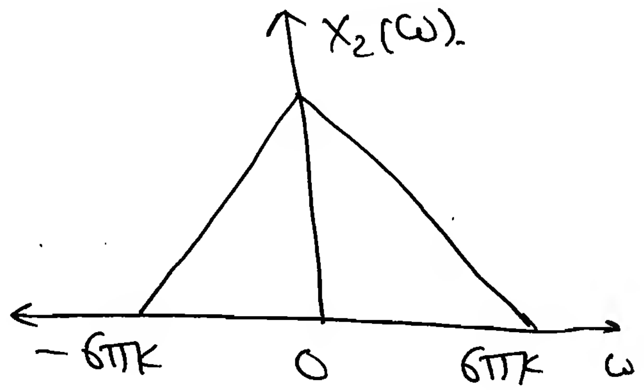


$\Rightarrow$ Max. freq. after addition of $X_1(\omega)$ & $X_2(\omega)$ is $6\pi K$ rad/sec.

$\Rightarrow \quad \omega_m' = 6\pi k$ rad/sec

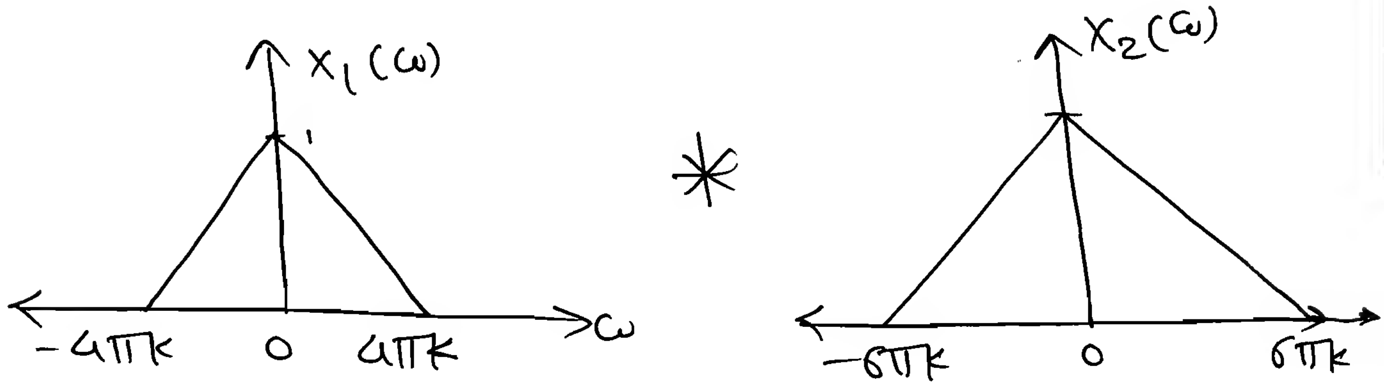
N.R. $\Rightarrow \quad \omega_0' = 2\omega_m' = 12\pi k$ rad/sec.

$\therefore \quad \boxed{f_s' = 2 f_m' = 6k\ Hz.}$

(d) $x_1(t) \cdot x_2(t)$.

Soln: Mul. $\xleftrightarrow{F.T.}$ Conv.

$\therefore \quad x_1(t) \cdot x_2(t) \xleftrightarrow{F.T.} X_1(\omega) * X_2(\omega)$.



$\Rightarrow$ After convolution new upper & lower limit of the signal are.

Lower limit = {Sum of the lower limit of $x_1(\omega)$ & $x_2(\omega)$}.

Upper limit = {Sum of the upper limit of $x_1(\omega)$ & $x_2(\omega)$}.

$\Rightarrow$ So, Lower limit = $\{-10\pi k\}$.

upper limit = $\{+10\pi k\}$.

So, $\omega_m' = 10\pi k$ rad/sec.

N.R. $\quad \omega_0' = 2\omega_m' = 2(10\pi k) = 20\pi k$ rad/sec.

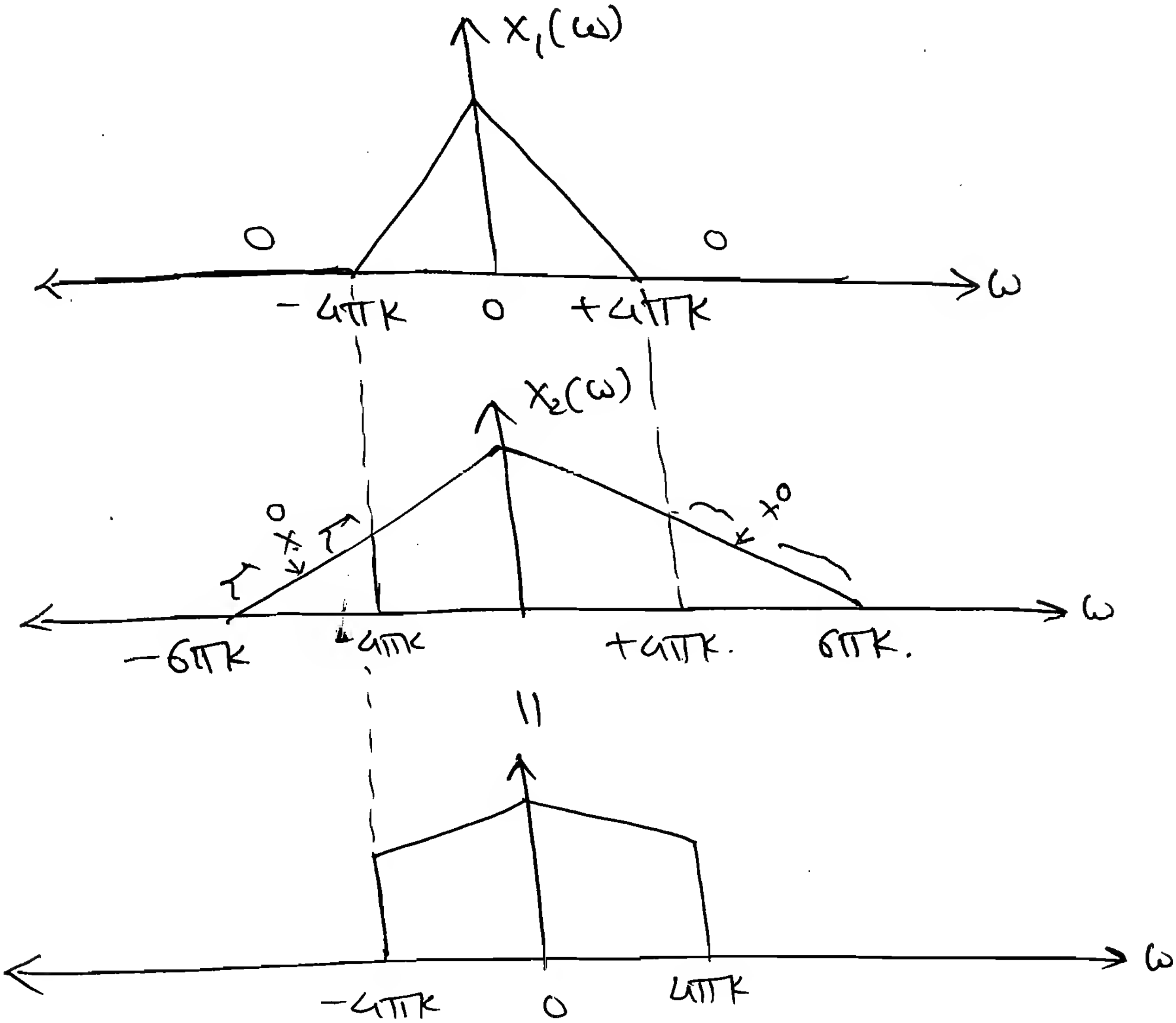
$$\Rightarrow \quad \boxed{f_s' = 2f_m = 10\text{ KHz.}}$$

(e) $x_1(t) * x_2(t)$.

Sol^n:

Conv. $\xleftrightarrow{\text{F.T.}}$ mult.

$\Rightarrow x_1(t) * x_2(t) \xleftrightarrow{\text{F.T.}} X_1(\omega) . X_2(\omega)$.



$\Rightarrow$ After Multiplication of two signal $X_1(\omega)$ & $X_2(\omega)$ New highest freq. is $4\pi K$ rad/sec.

So, N.R. $\Rightarrow$ $\omega_0' = 2\omega_m' = 2(4\pi K)$.

$= 8\pi K$ rad/sec.

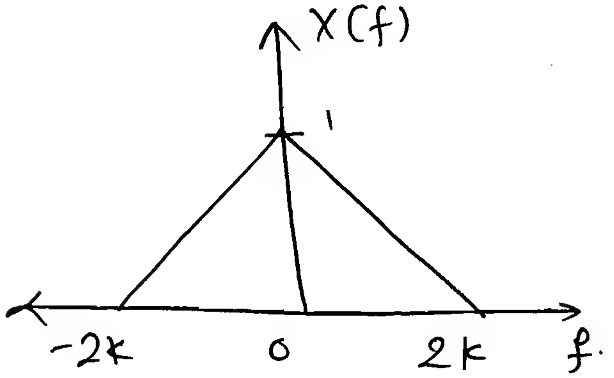
$$\boxed{f_s' = 2f_m = 4\text{ KHz.}}$$

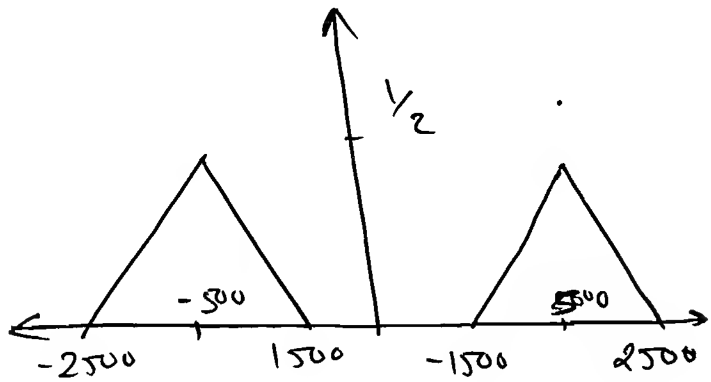
(f) $x_1(t) \cdot \cos(1000\pi t)$. $\rightarrow f_0 = 500$ Hz.

Soln:

$= x_1(t) \cdot \cos(1000\pi t)$.

$$\xleftrightarrow{F.T.} \frac{X_1(f - f_0) + X_1(f + f_0)}{2}$$





$\Rightarrow$ $f_m' = 2500$ Hz.

N.R. $2f_m = f_s = 5000$ H.

$\therefore$ $\boxed{f_s = 5\text{KHz}.}$

Note:-

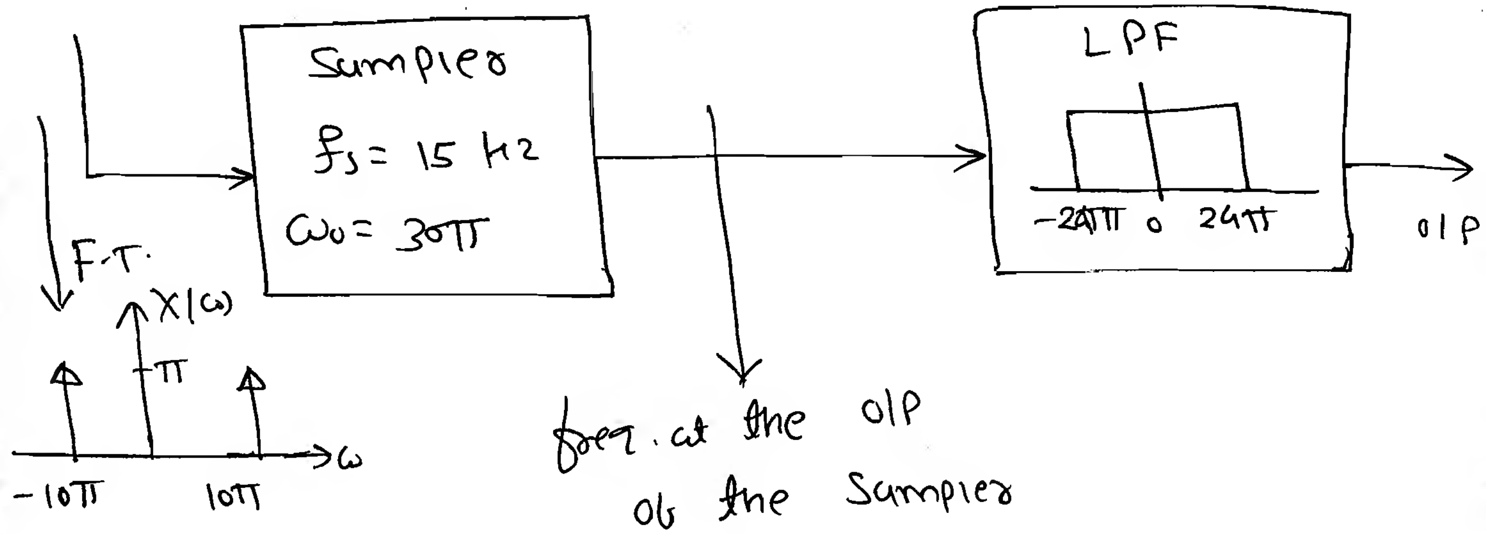
			B.W.
$x_1(t)$	$\longleftrightarrow$	$X_1(\omega)$ $\rightarrow$	ω_{m1}
$x_2(t)$	$\longleftrightarrow$	$X_2(\omega)$ $\rightarrow$	ω_{m2}.

	New BW.
① $x_1(t) \pm x_2(t) \xleftrightarrow{F.T.} X_1(\omega) \pm X_2(\omega)$	Max $(\omega_{m_1}, \omega_{m_2})$.
② $x_1(t) \cdot x_2(t) \longleftrightarrow X_1(\omega) * X_2(\omega)$	$(\omega_{m_1} + \omega_{m_2})$
③ $x_1(t) * x_2(t) \longleftrightarrow X_1(\omega) \cdot X_2(\omega)$.	Min $(\omega_{m_1}, \omega_{m_2})$.

Q A signal $x(t) = \cos(10\pi t)$ is a sampled at 15 Hz and is passed through an ideal L.P.F with cut of freq 12 Hz. What freq. will appear at the o/p of the filter?

Soln:

$x(t) = \cos(10\pi t)$



freq. at the o/p of the sampler

$(\omega - n\omega_0)$ $n \to -\infty$ to $+\infty$.

$(\omega_m - n\omega_0) \Rightarrow (f_m - nf_s)$

$\Rightarrow$ here. $\omega_m = 10\pi$

$\omega_0 = 30\pi$.

let, $n=0 \Rightarrow \omega = \pm 10\pi$ o/p

$n=1 \Rightarrow \omega = \omega_m - \omega_0$ → -20π (o/p), -40π

$n=-1 \Rightarrow \omega = \omega_m + \omega_0$ → 20π (o/p), 40π

$n=2 \Rightarrow \omega = \omega_m - 2\omega_0$ → -50π, -70π

So, freq. at LPF i.e at o/p of o/p $\Rightarrow \pm 10\pi, \pm 20\pi$.

P4.6.4 A signal represented by $x(t) = 5\cos(400\pi t)$ is sampled at a rate of 300 Hz. The resulting samples are passed through an ideal LPF with cut-off freq. of 150 Hz. Which of the following will be contained in the o/p of LPF?

(a) 100 Hz (b) 100 Hz, 150 Hz

(c) 50 Hz, 100 Hz (d) 20, 100, 150 Hz.

Soln:

$\omega_m = 400\pi \Rightarrow f_m = \pm 200$ Hz.

$f_s = 300$ Hz.

$\Rightarrow$ Freq. at o/p of the samples is,

$f = f_m - n f_s$, $n \to -\infty$ to $+\infty$.

$n=0$. $f = f_m = \pm 200$ Hz.

$n=1$ $f = f_m - f_s = 100$ Hz, -500 Hz.

$n=-1$ $f = f_m + f_s = 500$ Hz, 100 Hz

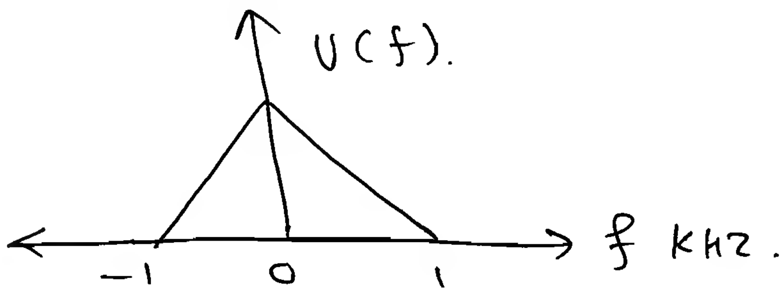
$n=2$ $f = f_m - 2f_s = 800$ Hz, -400 Hz.

$\Rightarrow$ Cut-off freq. of LPF is 150 Hz.

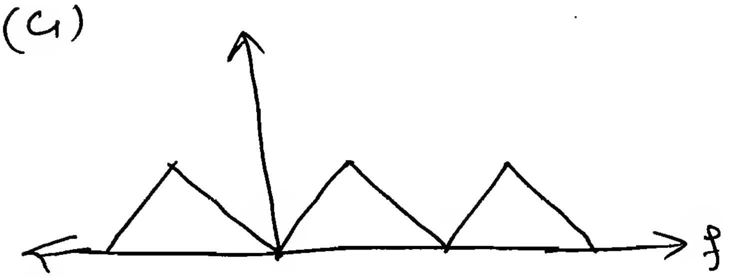
So, o/p freq. at LPF is in betn -150 to 150.

i.e ± 100 Hz So, ans - $\boxed{(a)\ 100\text{ Hz}}$

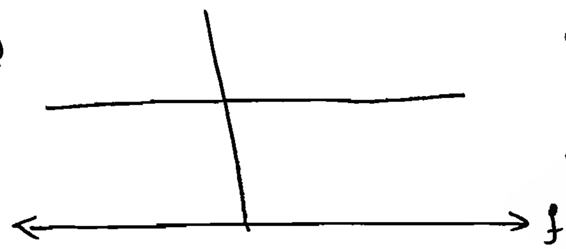
P4.6.5 The freq. Spectrum of a signal is shown in figure if this signal is ideally sampled at intervals of 1 msec, then the freq. spectrum of the sampled signal will be



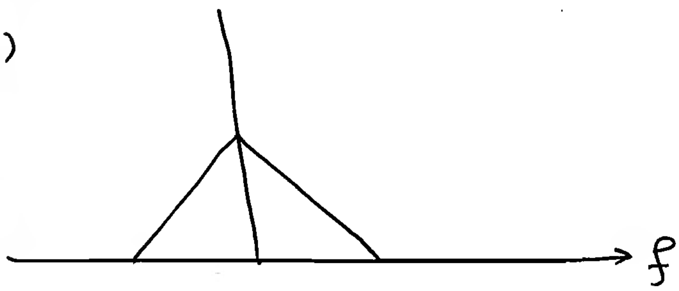
(a)



(b)



(c)



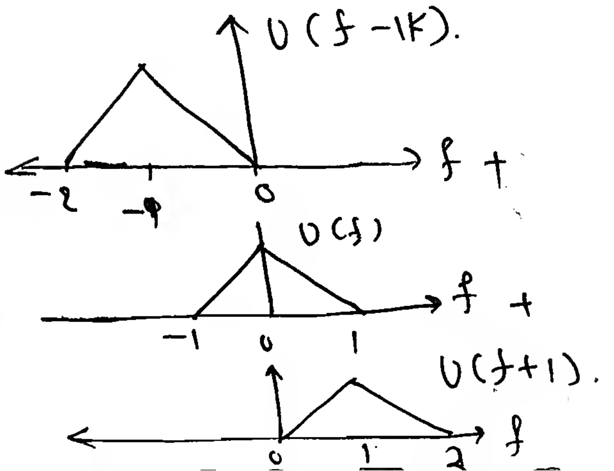
(d)

Soln:

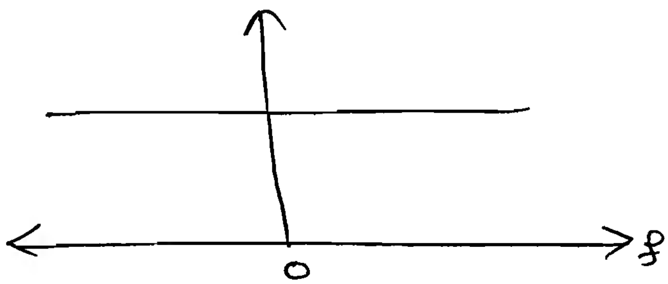
$f_m = 1$ KHz.

$f_s = 1$ msec.

$\therefore$ $f_s < 2 f_m$. So, under sampling.

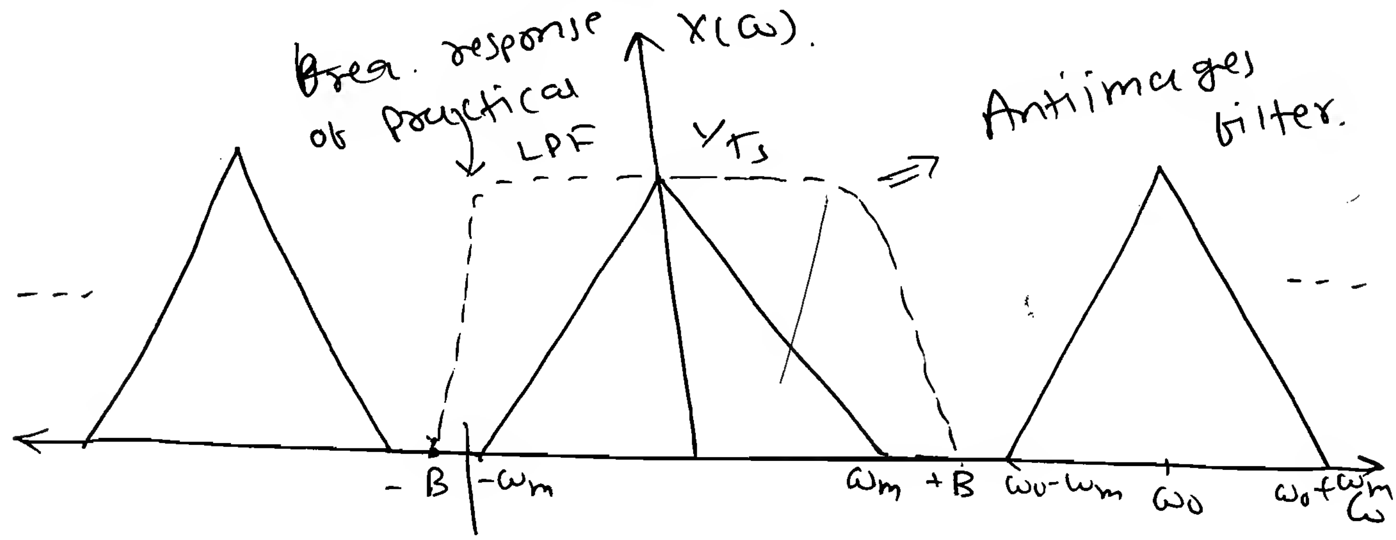


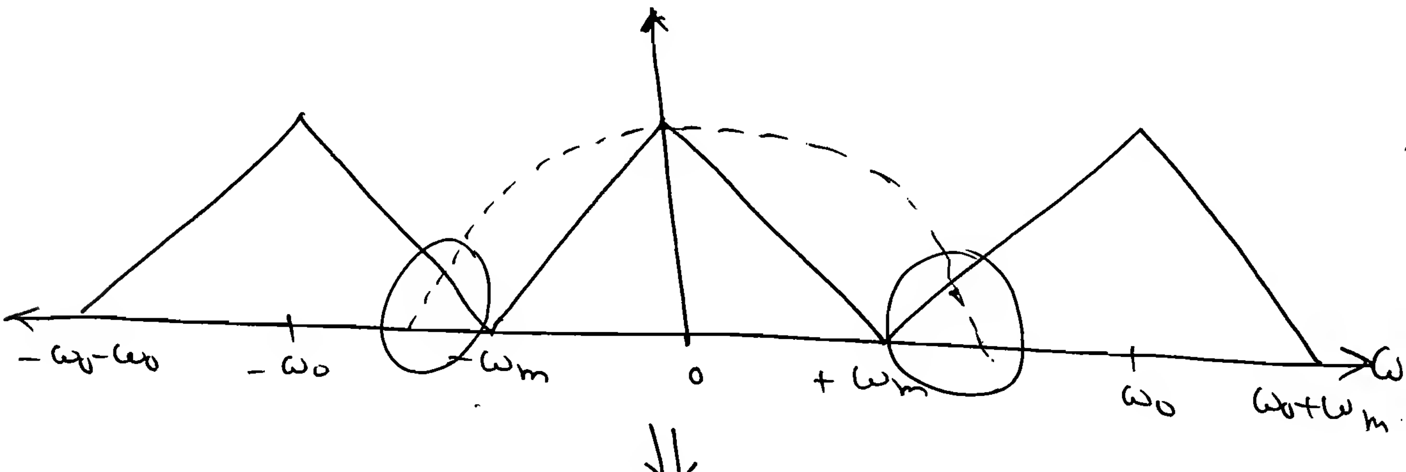
$\Rightarrow$



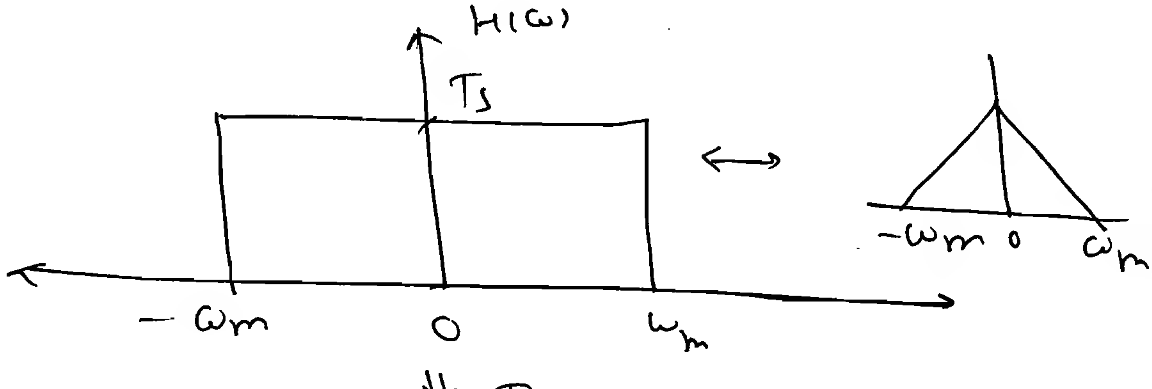
* Signal Reconstruction :-

⇒ The process of recovering original continous spectrum from the sampled spectrum is signal reconstruction.





⇓



⇓ Ideal LPF

⇒ $$\boxed{\omega_m < B < \omega_0 - \omega_m}$$

$\boxed{P\ 4.6.7}$ A signal $x(t) = 6\cos 10\pi t$ is sampled at a rate of 14 Hz to recover the original signal, cut-off freq. of the LPF should be ______.

(a) $5 < f_c < 9$ (b) 9 (c) 10 (d) 14.

Soln:

$\omega_m = 10\pi \Rightarrow f_m = 5\text{ Hz}$, $f_s = 14\text{ Hz}$

$\therefore$ $\boxed{f_m < B < f_s - f_m.}$

$\therefore$

$\therefore$ $5 < B < 14 - 5$

$\Rightarrow \boxed{5 < B < 9}$.

$\boxed{P\ 4.6.6}$ A signal with 2 freq. components at 6 kHz and 12 kHz is sampled at the rate of 16 kHz and then passed through a LPF having a cut-off freq. of 16 kHz. The output signal of the filter is ___.

(a) is an undistorted version of original signal.

(b) contains 6 kHz & spurious components of 4 kHz.

(c) contains only 6 kHz components.

(d) contains both components of original signal and 2 spurious components of

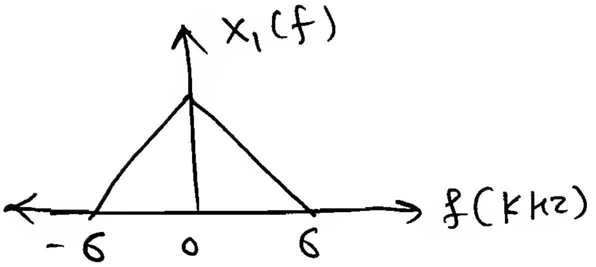
of 4 KHz & 10 KHz.

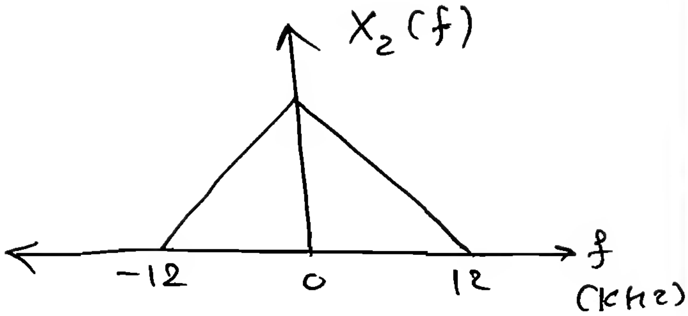
Soln:

$f_{m1} = 6$ KHz, $f_{m2} = 12$ KHz.

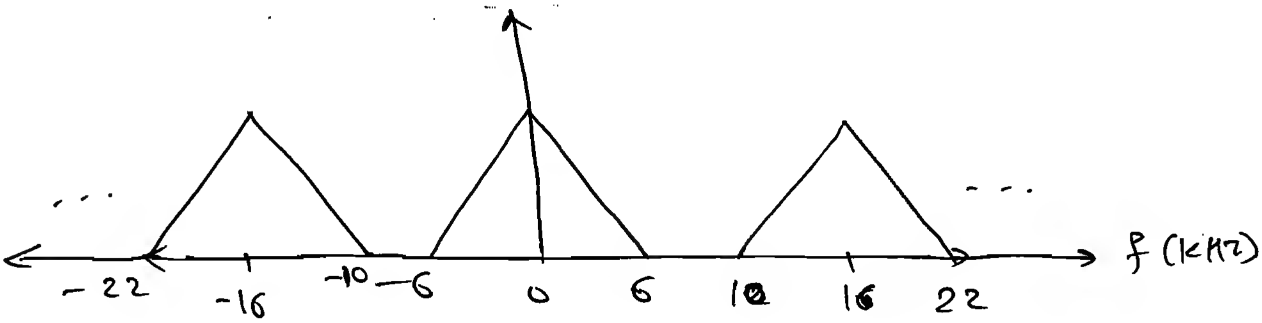
$f_s = 16$ KHz.

=>

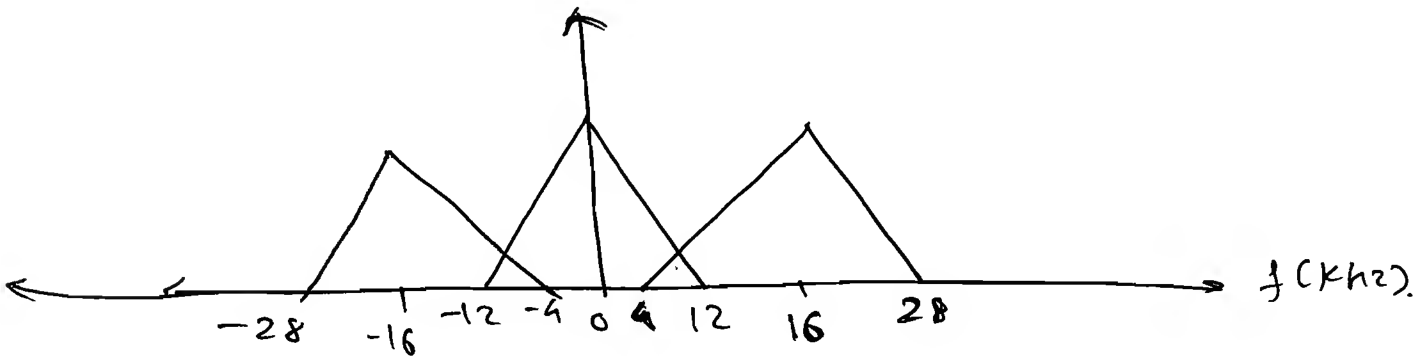


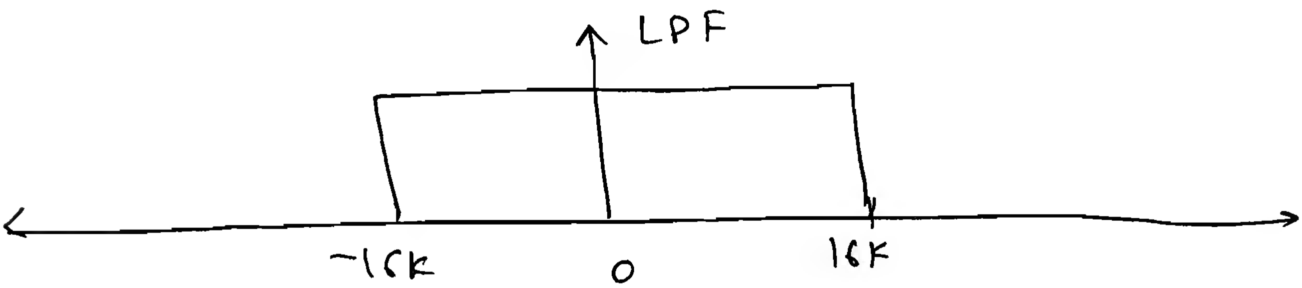


① $(f_s = 16K) > (2 f_{m1} = 12 KHz)$ => over sampling



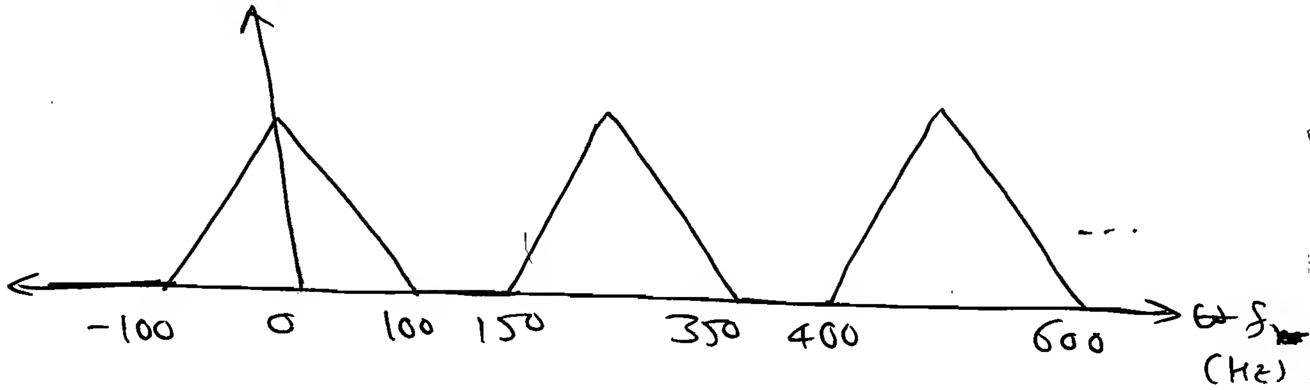
② $(f_s = 16K) < (2 f_{m2} = 24 KHz)$. => under sampling





=> Ans-(b) contains 6 KHz & spurious components of 4 KHz.

P 4.6.8 The Spectrum of a bandlimited signal after sampling is shown in figure. The value of sampling interval is ______.



Sol^n:

$f_m = 100$ Hz.

$\therefore f_s - f_m = 150$ Hz.

$\therefore f_s = 150 + 100 = 250$ Hz.

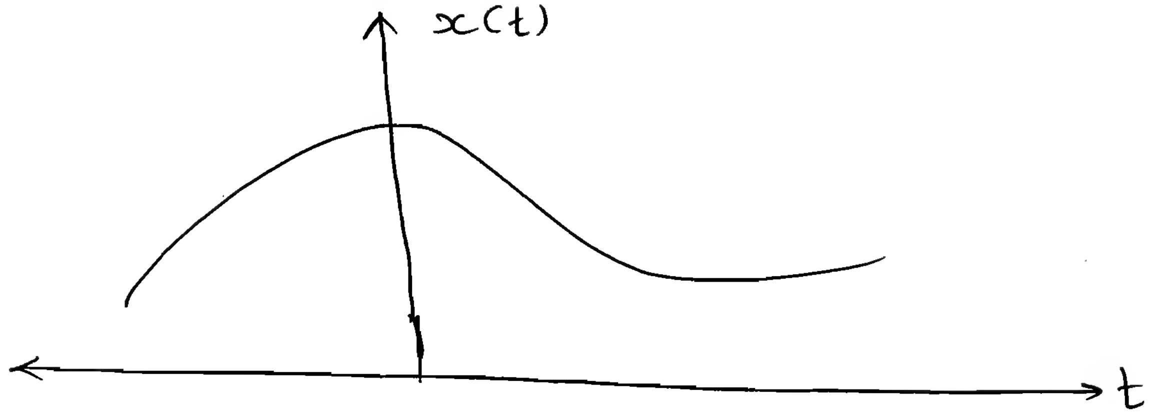
$\therefore f_s = 250$ Hz.

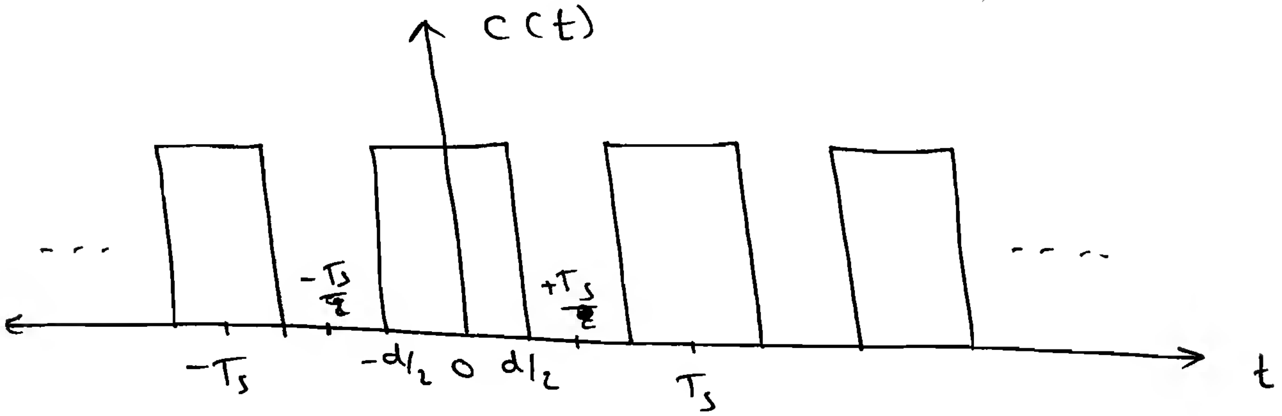
$\Rightarrow$ S.I. $T_s = \frac{1}{f_s} = \frac{1}{250} = 4$ m sec.

$\boxed{T_s = 4\,msec}$

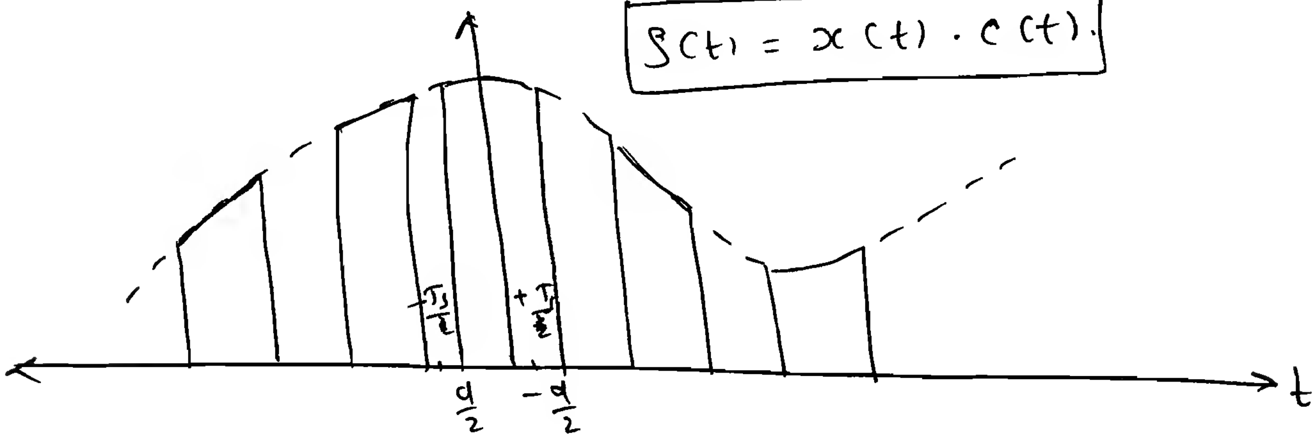
Note: The freq. at the o/p of sampler is $\omega - n\omega_0$ in ideal sampling. We can take the value of n from $-\infty$ to $+\infty$, but in natural sampling n value is desided by C_n.

* 2 Natural Sampling:





$$\boxed{s(t) = x(t) \cdot c(t).}$$



$$s(t) = x(t) \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}.$$

$\Big\downarrow$ F.T.

$$S(\omega) = \sum_{n=-\infty}^{\infty} C_n X(\omega - n\omega_0).$$

$$C_n = \frac{1}{T} \int_{-d/2}^{d/2} (1). e^{-jn\omega_0 t} dt$$

$\Rightarrow \quad T = T_s.$

$$\therefore \; C_n = \frac{1}{2T_s} \times \left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right]_{-d/2}^{+d/2}$$

$$= \frac{1}{T_s} \times \left[\frac{e^{-jn\omega_0 \frac{d}{2}} - e^{jn\omega_0 \frac{d}{2}}}{-jn\omega_0} \right].$$

$$C_n = \frac{\sin\left(\frac{n\omega_0 d}{2}\right)}{n\omega_0 T_s}.$$

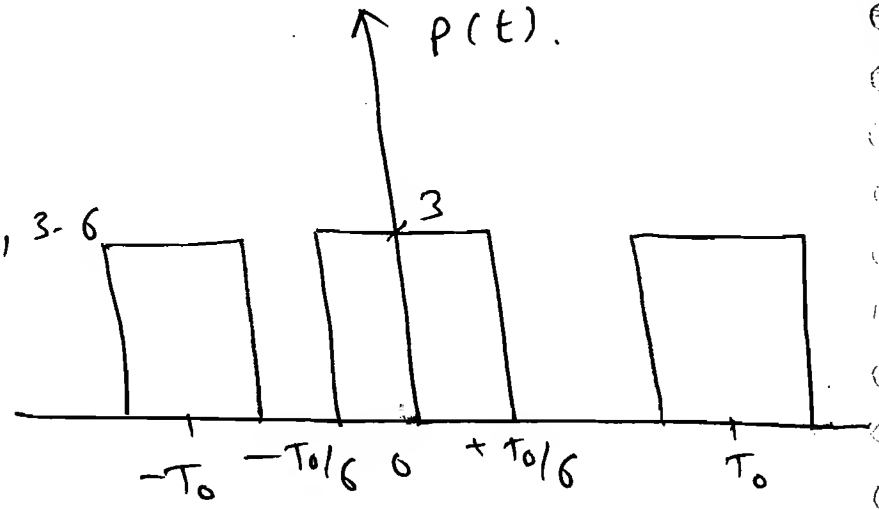
$\boxed{\text{P 4.6.9}}$ Let $x(t) = 2\cos(800\pi t) + \cos(1400\pi t)$ and $x(t)$ is sampled with the rectangular pulse train as shown in fig. The only spectral components (in KHz) in the sampled signal in the freq. range 2.5 KHz to 3.5 KHz.

(a) 2.7, 3.4

(b) 3.3, 3.6

(c) 2.6, 2.7, 3.3, 3.4, 3.6

(d) 2.7, 3.3.



Soln:

$$C_n = \frac{1}{T_0} \int_{-T_0/6}^{T_0/6} (3).\, e^{-jn\omega_0 t} \, dt$$

$\omega_0 = \frac{2\pi}{2T_0}$

$\omega_0 = 2\pi$ K rad/sec

$f_s = \frac{1}{2}$ KHz.

$$C_n = \frac{\sin\left(\frac{n\omega_0 d}{2}\right)}{n\omega_0 T_s}$$

$$= \frac{\sin\left(\frac{n}{2} \times \frac{2\pi}{2\times10^{-3}} \times \frac{10^{-3}}{6}\right)}{n \times \frac{2\pi}{2T_s} \times T_s}$$

$$C_n = \frac{\sin\left(\frac{n\pi}{3}\right)}{n\pi}$$

$C_n = 0$ for $n = 3, 6, 9, \ldots$

$C_n \neq 0$ for $n = 0, 1, 2, 4, 5, \ldots$

⇒ freq. at the o/p of natural sampler.

$$\left.\begin{array}{l} f_{m_1} \pm n f_s \\ f_{m_2} \pm n f_s \end{array}\right] \quad n = 0, 1, 2, 4, 5, 7, 8, \ldots$$

⇒ $n = 0$ ⇒ $f_{m_1} = f = 0.4k$

$f_{m_2} = f = 0.7k.$

⇒ $n = 1$ ⇒ $f_1 = f_{m_1} \pm f_s = 0.4 + 1k = 1.4\ kHz$

$f_2 = f_{m_2} \pm f_s = 0.7 + 1k = 1.7\ kHz.$

⇒ $n = 2$ ⇒ $f_1 = f_{m_1} \pm 2f_s = 0.4 + 2k = 2.4\ kHz$

$f_2 = f_{m_2} + 2f_s = 0.7 + 2k = 2.7\ kHz.$

⇒ $n = 4$ ⇒ $f_1 = f_{m_1} \pm 4f_s = 0.4 \mp 4k = 3.6\ k$

$f_2 = f_{m_2} \mp 4f_s = 0.7 \mp 4k = 3.3\ k.$

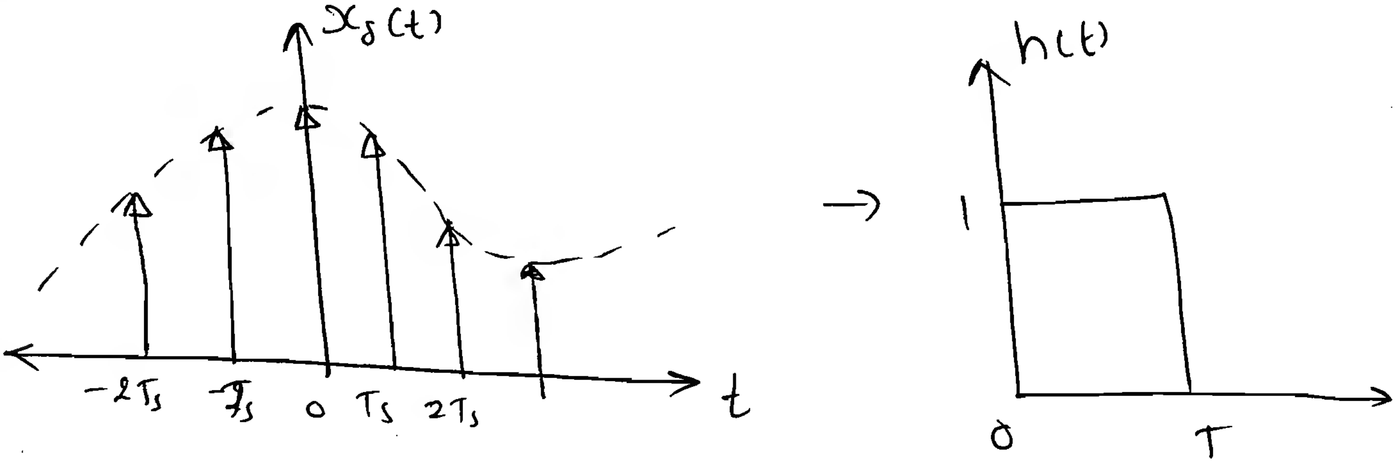
⇒ ~~$n = -1$ ⇒ $f_1 = f_{m_1} - f_s$~~

~~$f_2 = f_{m_2} - f_s$~~

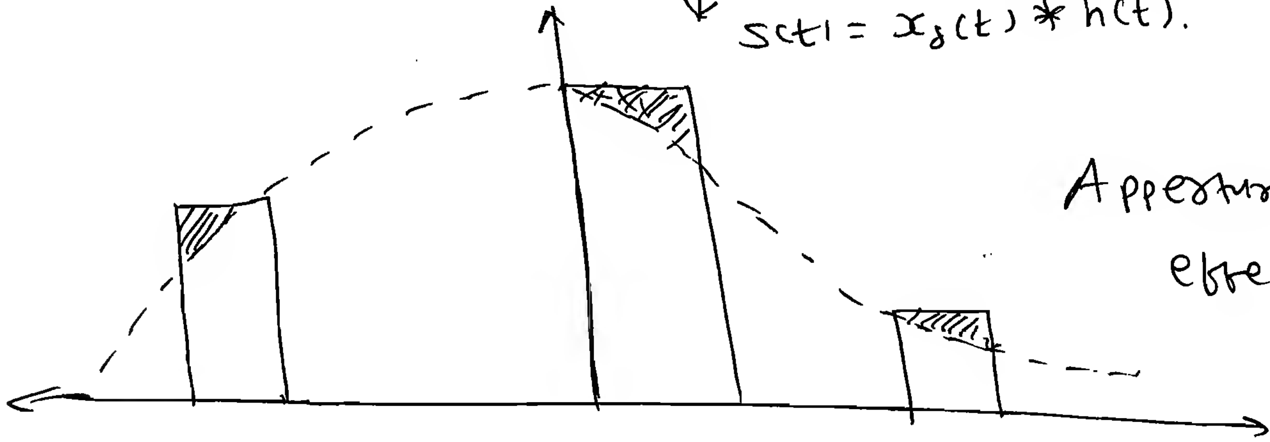
So, Ans: (d) 2.7, 3.3.

③ Flat - Top Sampling:

⇒

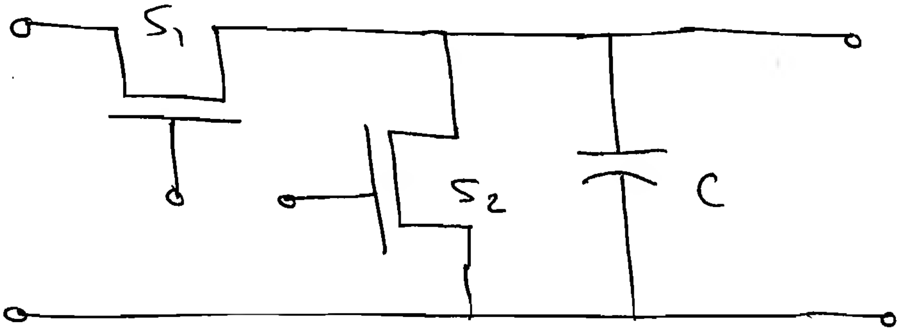


$s(t) = x_\delta(t) * h(t).$



Apperture effect.

⇒

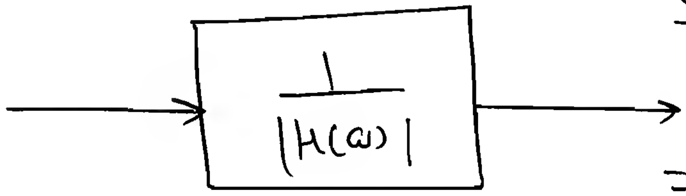


Sample and hold ckt.

⇒ because of maintaining constant amplitude level we introduce amplitude distortion of $T\,\text{sinc}\left(\frac{\omega T}{2\pi}\right)$ & phase Delay of $-\frac{\omega T}{2}$, which is known as Apperture effect.

$\Rightarrow$ To cancel this, Flat-Top sampled sig. is applied to an equalizer $\frac{1}{|h(\omega)|}$.

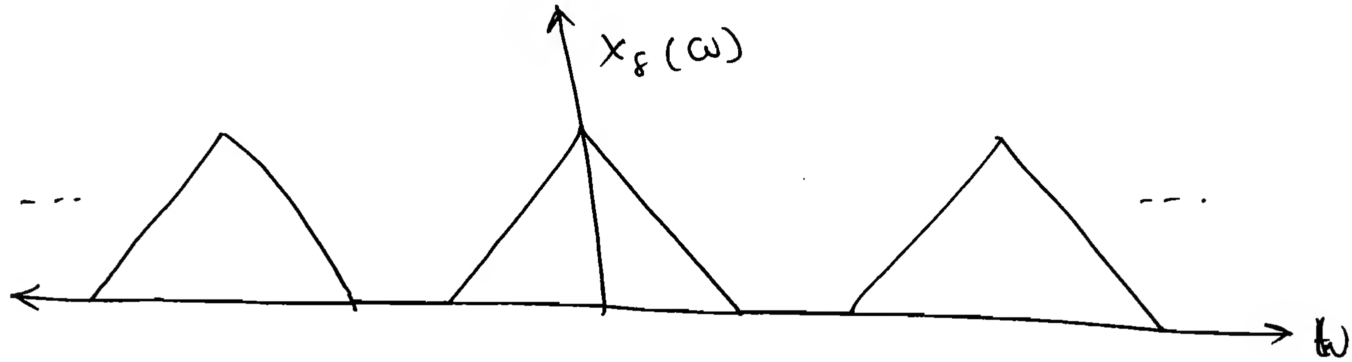
$\Rightarrow$



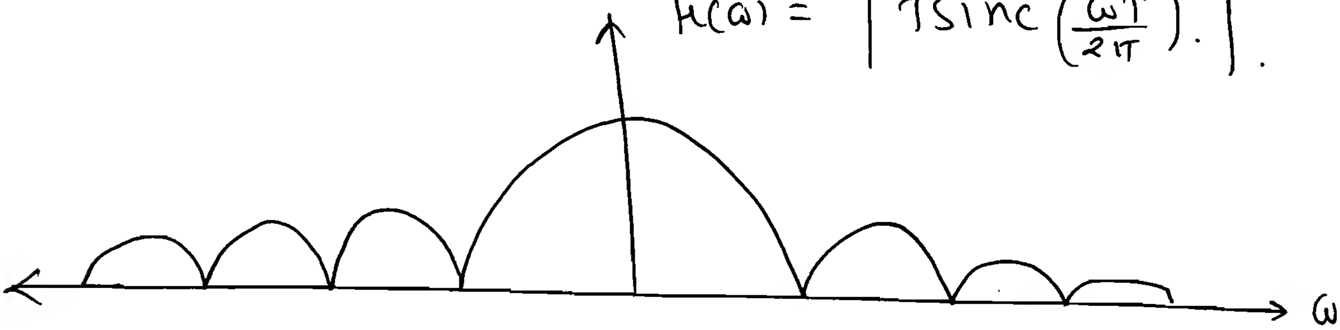
$$S(\omega) = X_\delta(\omega) \cdot H(\omega)$$

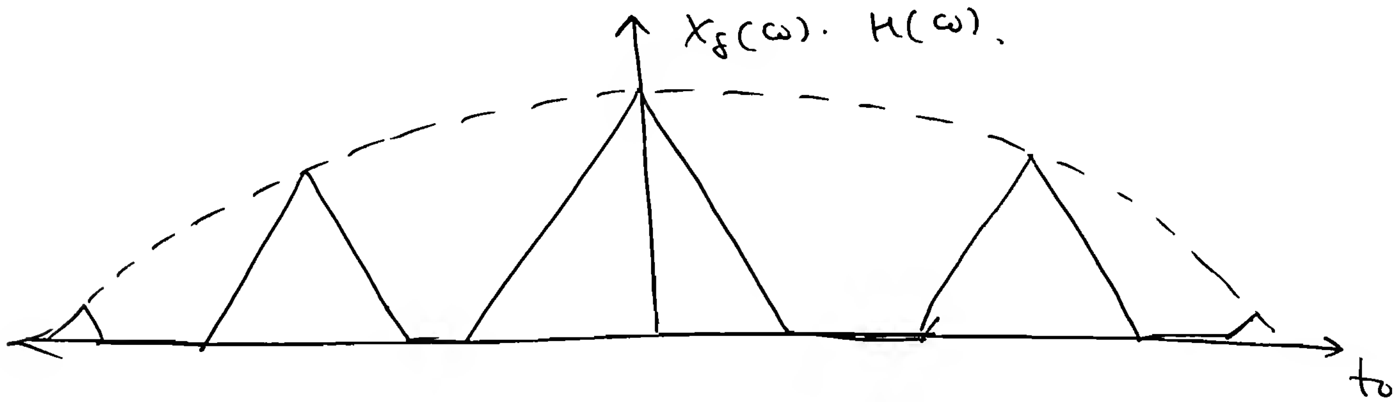
$$= \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X(\omega - n\omega_0) \cdot H(\omega).$$

Equalizer $\quad H(\omega) = T\,\text{sinc}\left(\frac{\omega T}{2\pi}\right) \cdot e^{-j\omega(T/2)}.$

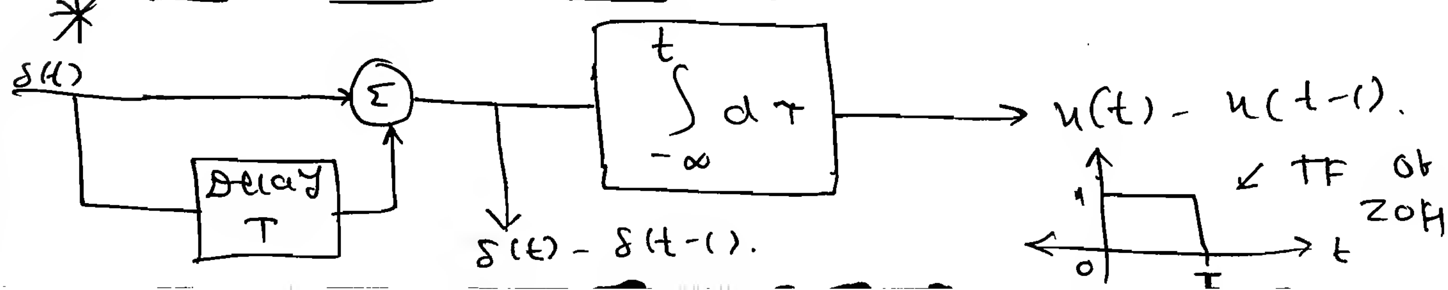


$H(\omega) = \left| T\,\text{sinc}\left(\frac{\omega T}{2\pi}\right). \right|.$





* Zero order Hold (ZOH) ckt.



Ch-5 - Laplace Transform :-

Purpose:- All differential and integrals are converted to simple algebric eqn.

⇒ L.T. expresses signals as linear combination of complex exponentials, which are eigen functions of DE which describe continuous - time LTI systems.

⇒ The primary role of the L.T in engineering is the transient & stability analysis of causal LTI systems.

⇒ In addition to its simplicity, many design techniques in circuits, filters & control systems have been developed in L.T. domain.

⇒ Generalization of F.T. is laplace Trans.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

⇒ Let, Input $x(t) = e^{st}$ $(s = \sigma + j\omega)$

⇓

Complex Variable

$\Rightarrow$ O/p is $y(t) = e^{st} \cdot H(s)$.

$\rightarrow y(t) = x(t) * h(t)$.

$$= \int_{-\infty}^{+\infty} x(t-\tau) \cdot h(\tau) d\tau.$$

$$= \int_{-\infty}^{+\infty} e^{s(t-\tau)} \, h(\tau) \cdot d\tau.$$

$$y(t) = e^{st} \cdot \int_{-\infty}^{+\infty} e^{-s\tau} \, h(\tau) \cdot d\tau.$$

$$\therefore \boxed{y(t) = e^{st} \cdot H(s)}$$

$\Rightarrow$ L.T. of general signal $x(t)$

$$\boxed{L[x(t)] = \int_{-\infty}^{+\infty} x(t) \cdot e^{-st} \, dt} = X(s).$$

$s = \sigma + j\omega$.

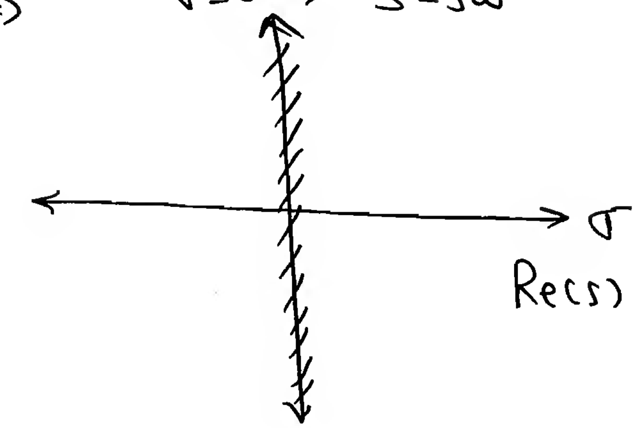
$$\Rightarrow X(\sigma + j\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-(\sigma + j\omega)t} \, dt.$$

$$= \int_{-\infty}^{+\infty} \left[x(t) \cdot e^{-\sigma t} \right] \cdot e^{-j\omega t} \, dt$$

$$\therefore \boxed{L[x(t)] = F\{x(t) \cdot e^{-\sigma t}\}}.$$

$\Rightarrow$ $e^{-\sigma t}$ may be decaying (or) growing depending on whether 'σ' is +ve (or) -ve.

$\Rightarrow$ $\sigma = 0 \Rightarrow s = j\omega$



L.T.

- BLT $\Downarrow$ causality, stability, freq. response
- U.L.T. $\Downarrow$ used to solve D.E. with I.C.S.

* Region of Convergence of L.T. : (ROC)

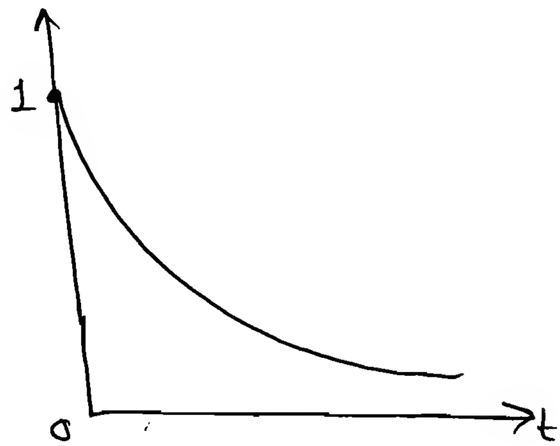
$$X(s) < \infty.$$

$$\Rightarrow \int_{-\infty}^{+\infty} \left| x(t) \cdot e^{-\sigma t} \right| dt < \infty.$$

Necessary.

* L.T. of Standard Signals:

① $x_1(t) = e^{-at} u(t)$; $\mathrm{Real}\{a\} > 0$.



$$\Rightarrow X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} \cdot dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-st} \cdot dt.$$

$$= \int_{0}^{\infty} e^{-(s+a)t} \cdot dt.$$

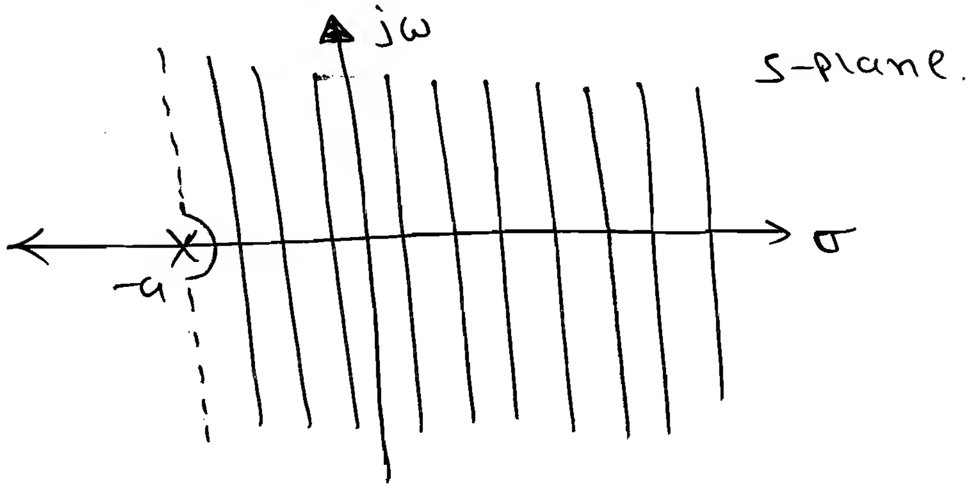
$$= \left[\frac{e^{-(s+a)t}}{-(s+a)} \right]_0^{\infty}.$$

$$= \frac{1}{\infty} + \frac{1}{s+a}$$

$$= \frac{1}{s+a} \quad ; \quad \sigma + a > 0$$

$$\underbrace{\sigma > Re\{-a\}.}_{R.O.C.}$$

$$\rightarrow \quad e^{-at} \cdot u(t) \longleftrightarrow \frac{1}{s+a} \quad ; \quad Re(s) > -a.$$



~~2) x(t) = -e^{at} u(-t).~~

$\Rightarrow$ R.O.C. of LT consist of lines llel to jω- axis.

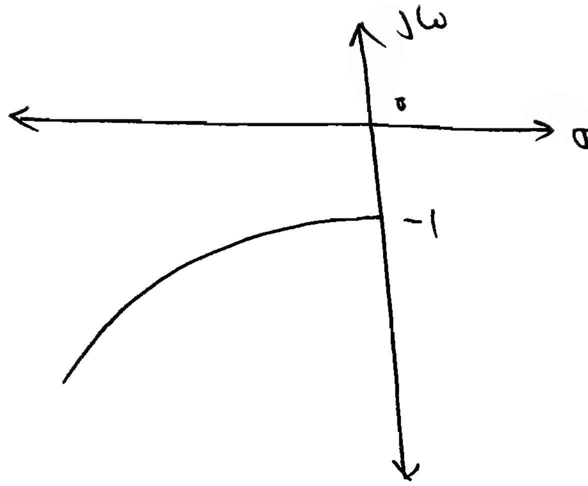
$\Rightarrow$ ROC do not contain any Pole.

$\Rightarrow$ For Stability in laplace domain ROC must include Imaginary axis.

(2) $x_2(t) = -e^{-at} u(-t)$; $Re\{a\} > 0$.

$$\Rightarrow \quad x_2(t) = 0 \; ; \quad t > 0$$
$$= -e^{-at} \; ; \quad t < 0.$$

$$\Rightarrow X_2(s) = \int_{-\infty}^{0} -e^{-at} \cdot e^{-st} \cdot dt$$



$$= \int_{-\infty}^{0} -e^{-(s+a)t} \cdot dt$$

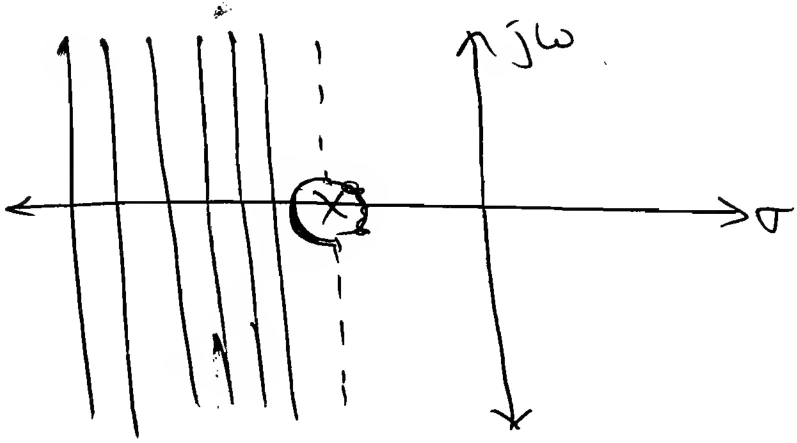
$$= \left[\frac{e^{-(s+a)t}}{(s+a)}\right]_{-\infty}^{0} = \left[\frac{e^{-(\underline{\sigma+a}+j\omega)t}}{s+a}\right]_{-\infty}^{0}$$

$$X_2(s) = \frac{1}{s+a} \; ; \quad \sigma + a < 0. \quad , \quad Re(s) < -a$$

$$Re(\sigma + j\omega) < -a$$

$$\Rightarrow \boxed{\sigma < -a.}$$

$\Rightarrow$

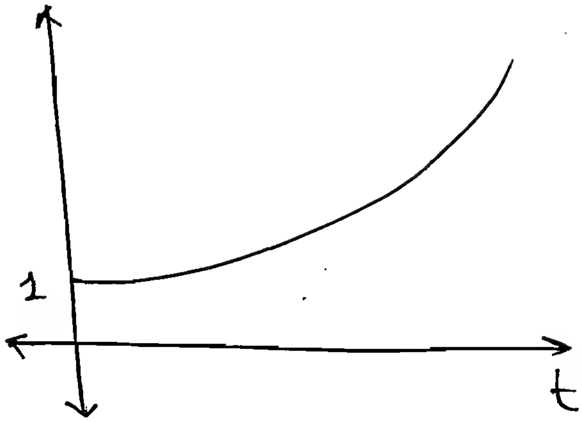


$$\Rightarrow \quad e^{-at} u(t) \longleftrightarrow \frac{1}{s+a} \; ; \quad \sigma > Re\{-a\}.$$

$$-e^{-at} \cdot u(-t) \longleftrightarrow \frac{1}{s+a} \; ; \quad \sigma < Re\{-a\}.$$

$\Rightarrow$ Note: Solution of Laplace transform is unique only when the RoC is given.

③ $x_3(t) = e^{at}u(t) \quad ; \quad Re\{a\} > 0.$



$$\Rightarrow X_3(s) = \int_0^\infty e^{at} \cdot e^{-st} \cdot dt.$$

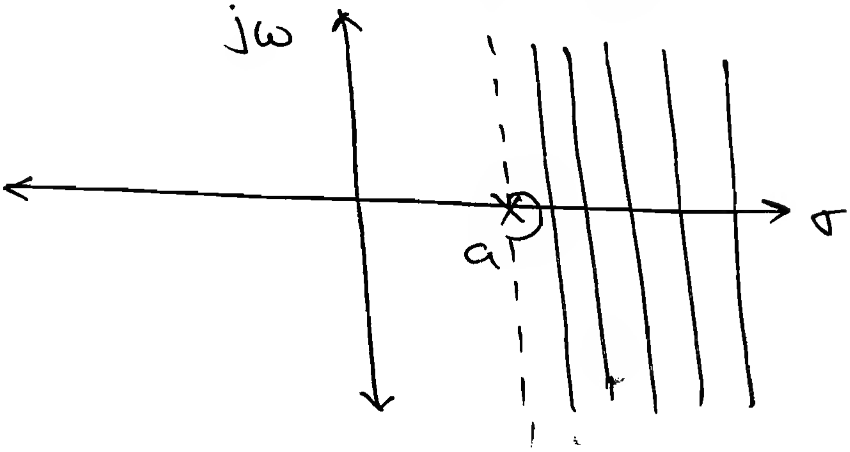
$$= \int_0^\infty e^{-(s-a)t} \cdot dt.$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty$$

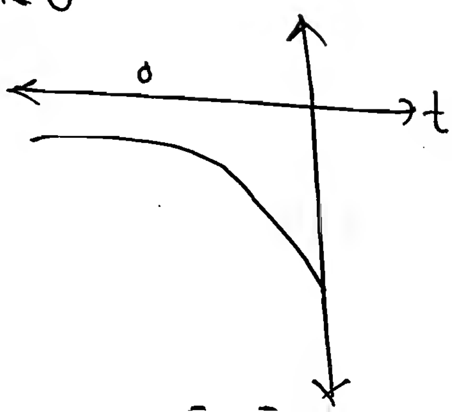
$$= \left[\frac{e^{-(\sigma-a+j\omega)t}}{-(s-a)}\right]_0^\infty.$$

$$\therefore X_3(s) = \frac{1}{(s-a)}; \quad \sigma - a > 0$$

$$\sigma > Re\{a\}.$$



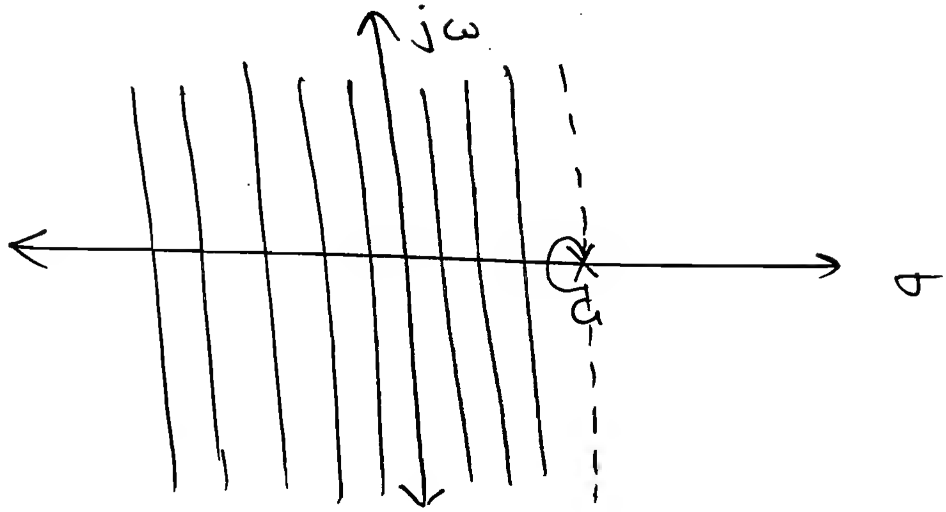
④ $X_4(t) = -e^{at}u(-t). \quad , \quad Re\{a\} \geq 0.$



$$\Rightarrow X_4(s) = \frac{1}{s-a}; \quad Re\{s\} < a.$$

$$\sigma < Re\{a\}$$

$$-e^{at}u(-t) \longleftrightarrow \frac{1}{s-a} \; ; \; \sigma < a.$$



=)

$$\boxed{\begin{array}{llll} e^{-at}u(t) & \longleftrightarrow & \dfrac{1}{s+a} & ; \; \sigma > Re\{-a\} \\ -e^{-at}u(-t) & \longleftrightarrow & \dfrac{1}{s+a} & ; \; \sigma \not> Re\{-a\}. \\ e^{at}u(t) & \longleftrightarrow & \dfrac{1}{s-a} & ; \; \sigma > Re\{a\}. \\ -e^{at}u(-t) & \longleftrightarrow & \dfrac{1}{s-a} & ; \; \sigma < Re\{a\}. \end{array}}$$

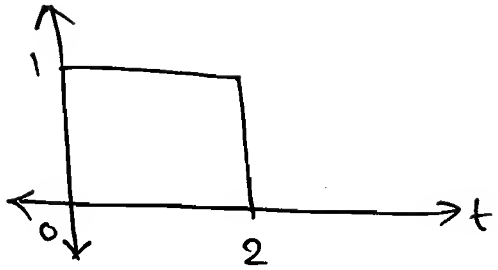
⇒ If $x(t)$ is a finite duration then the Roc is complete s-plane.

e.g. $x(t) = \delta(t)$.

$\downarrow$ L.T.

$X(s) = 1$, Roc entire s-plane.

e.g.



$x(t) = u(t) - u(t-2)$.

$X(s) = \frac{1}{s} - \frac{e^{-2s}}{s}$.

$$\lim_{s \to 0} \frac{1 - e^{-2s}}{s}$$

$$= \lim_{s \to 0} \frac{0 + 2e^{-2s}}{(1)} \quad (\because \text{L' hospital rule.}$$

$$= 2$$

Roc is entire s-plane.

* Properties of L.T.

(1.) Linearity :

$\Rightarrow$ If $x_1(t) \longleftrightarrow X_1(s)$ with $ROC = R_1$

$x_2(t) \longleftrightarrow X_2(s)$ with $ROC = R_2$.

then $a x_1(t) + b x_2(t) \longleftrightarrow a X_1(s) + b X_2(s)$

with $ROC = R_1 \cap R_2$.

P 5.1.2 Find the L.T. of the following signals with R.O.C.?

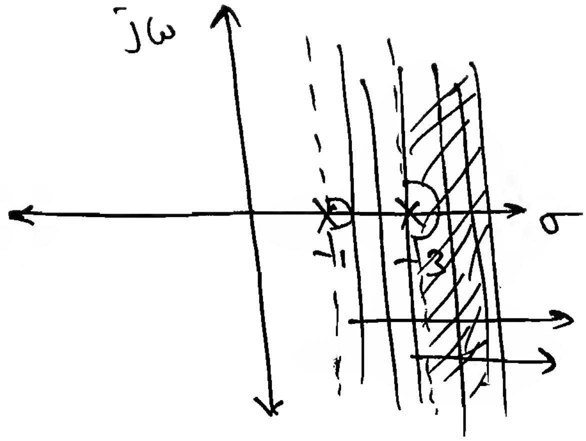
i) $x_1(t) = e^{-t} u(t) + e^{-3t} u(t)$.

Soln: $X_1(s) = \frac{1}{(s+1)} + \frac{1}{(s+3)}$

$\sigma > -1$ $\quad$ $\sigma > -3$. $\quad$ common ROC.

$\boxed{\sigma > -1}$

$\Rightarrow$



<u>Note</u> :- If the L.T. $X(s)$ of $x(t)$ is rational, then if $x(t)$ is right sided the ROC is the region in the s-plane to the right of the right most pole and if $x(t)$ is left sided, ROC is left of the left most pole.

→ In above case right of the right most pole is -3 so, ROC: $\sigma > -3$.

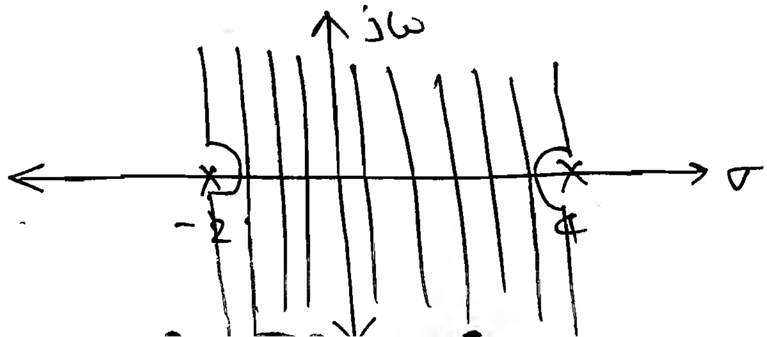
(2) $x_2(t) = e^{-2t}u(t) + e^{+4t}u(-t)$.

Soln: $X_2(s) = \dfrac{1}{(s+2)} - \dfrac{1}{(s-4)}$

$\uparrow \qquad\qquad \uparrow$

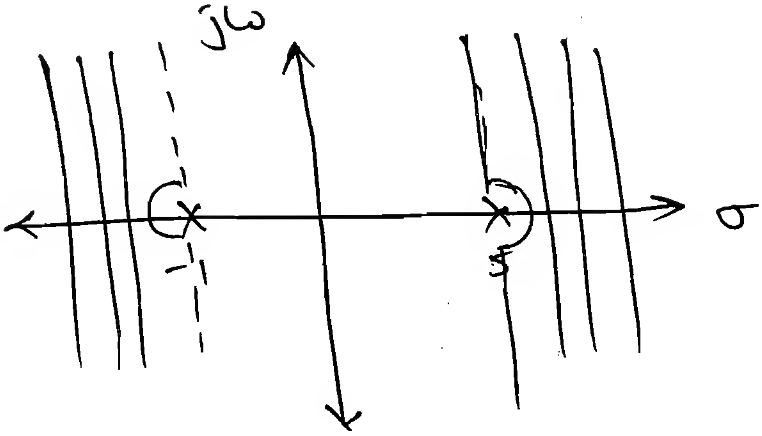
$\sigma > -2 \qquad \sigma < 4$.

So, ROC: $-2 < \sigma < 4$.



(3) $x_3(t) = e^{-t}u(-t) + e^{5t}u(t)$.

$\underline{\underline{\text{Sol}^n}}$: $X_3(s) = \underbrace{-\dfrac{1}{(s+1)}}_{\sigma < -1} + \underbrace{\dfrac{1}{(s-5)}}_{\sigma > 5}$



No Common ROC so L.T. Can not exist.

(4) $x_4(t) = 1 \quad \forall t$.

$\underline{\underline{\text{Sol}^n}}$: $x_4(t) = \underbrace{u(t)}_{\sigma > 0} + \underbrace{u(-t)}_{\sigma < 0}$.

No Common ROC. So L.T do not exist.

(5) $x_5(t) = \text{sgn}(t)$.

$\underline{\underline{\text{Sol}^n}}$: $x_5(t) = \underbrace{u(t)}_{\sigma > 0} - \underbrace{u(-t)}_{\sigma < 0}$

No Common ROC. So L.T. do not exist.

$\boxed{\text{PS-1.3}}$ Consider the signal $x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$ & its L.T. is $X(s)$. What are

Constraints placed on the real & imaginary parts of β if the ROC of $X(s)$ is $Re\{s\} > -3$?

Sol:

$$x(t) = e^{-5t}u(t) + e^{-\beta t}u(t).$$

$$X(s) = \frac{1}{(s+5)} + \frac{1}{(s+\beta)}.$$

$\sigma > -5$ $\qquad$ $\sigma > -\beta.$

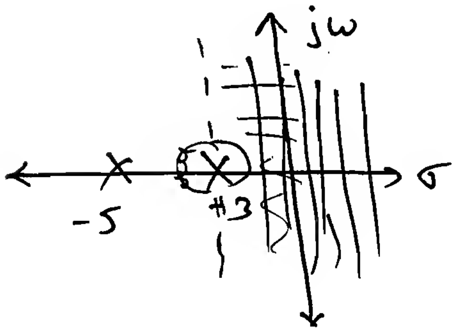
$$\boxed{\begin{array}{l}\sigma + Re\{\beta\} > 0 \\ \sigma + 3 > 0\end{array}}$$

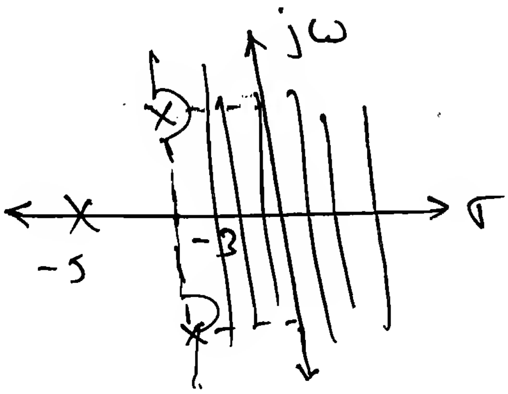
Now, ROC of $X(s)$ is $Re\{s\} > -3$

So, $\sigma > Re\{\beta\}.$

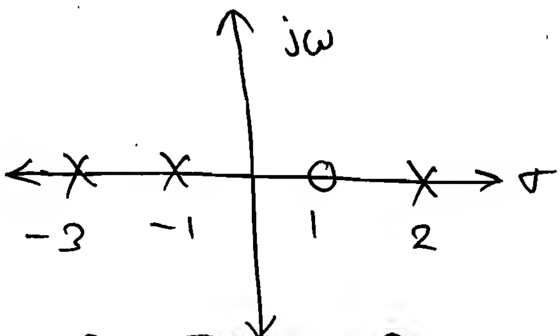
So, $\boxed{Re\{\beta\} = 3.}$

$Im\{\beta\} \Rightarrow$ any value.





$\boxed{P5.1.4}$ How many possible ROC are there for the pole-zero plot shown in fig (1)?



Sol^n: Possible RoC.

① $\sigma < -3$ ③ $-3 < \sigma < -1$.

② $\sigma > 2$ ④ $-1 < \sigma < 2$.

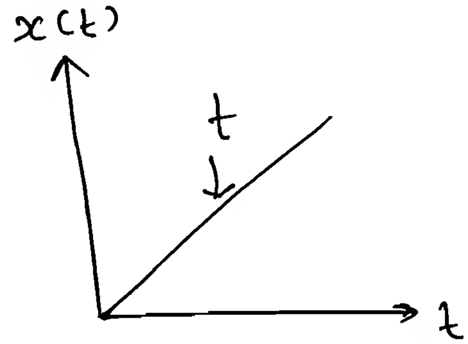
$\boxed{P\ 5.1.5}$ In what range should $Re\{s\}$ remain so that the L.T. of the f^n $e^{(a+2)t+5} \cdot u(t)$ exists?

Sol^n: $x(t) = e^{(a+2)t} \cdot e^{5} \cdot u(t)$.

so, ~~Re (a+2)~~ $\boxed{Re\{s\} > (a+2)}$

$\boxed{a}$ $x(t) = t \cdot u(t)$.

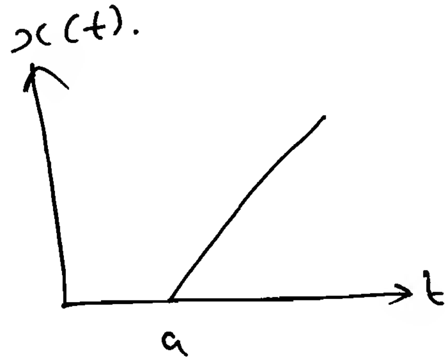
Sol^n: $X(s) = \dfrac{1}{s^2}$



$\boxed{a}$ $(t-a)\, u(t-a)$.

Sol^n: shifting Property.

$X(s) = \dfrac{e^{-as}}{s^2}$.



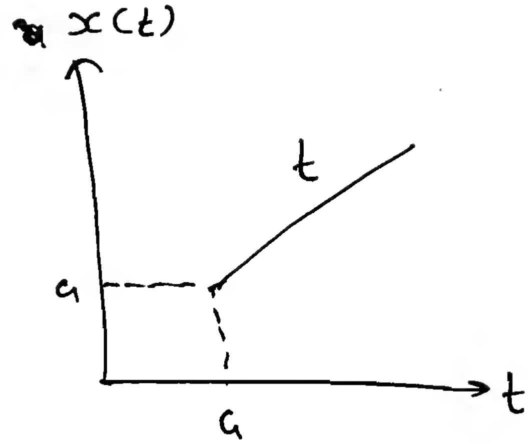
$\boxed{a}$ $(t-a)\, u(t)$.

Sol^n: $x(t) = (t-a)\, u(t)$

$= t \cdot u(t) - a u(t)$.

$$\Rightarrow \quad X(s) = \frac{1}{s^2} - \frac{a}{s}.$$

$\boxed{e}$ $\quad t\,u(t-a).$



Sol^n:

$$x(t) = t\,u(t-a).$$

$$\Rightarrow x(t) = [(t-a)+a]\,u(t-a).$$

$$= (t-a)\cdot u(t-a) + a\,u(t-a).$$

$\downarrow$ L.T.

$$X(s) = \frac{e^{-as}}{s^2} + \frac{a\cdot e^{-as}}{s^2}.$$

$$\boxed{X(s) = \frac{e^{-as}}{s^2}[1+sa].}$$

(or)

$$\Rightarrow \quad x(t) = t\,u(t-a).$$

$$L[x(t)] = L[t\,u(t-a)].$$

$$= e^{-as}\,L[t+a].$$

$$= e^{-as}\left[\frac{1}{s^2} + \frac{a}{s}\right].$$

$$\boxed{X(s) = \frac{e^{-as}}{s^2}[1+as]}$$

$$X(s) = \int_{-\infty}^{+\infty} x(t)\cdot e^{-st}\,dt.$$

$$X(s) = \text{F.T.}\{x(t)\cdot e^{-\sigma t}\}.$$

when $\sigma = 0 \Rightarrow s = j\omega$

$$X(s) = F.T.\{x(t)\}.$$

So, $\boxed{\text{L.T. of } x(t) \text{ calculated on } j\omega \text{ axis (i.e. } s=j\omega) \text{ is nothing but the F.T. of } x(t).}$

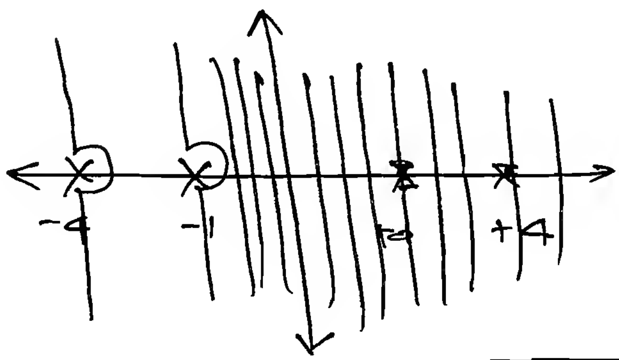
$\Rightarrow$ $\boxed{\begin{array}{l}\text{Right- sided} \Rightarrow u(t). \\ \text{Left- sided} \Rightarrow -u(-t).\end{array}}$ ***

e.g. $X(s) = \dfrac{1}{(s+1)(s+4)}$.

Sol^n^: $X(s) = \dfrac{1}{(s+1)(s+4)}$.

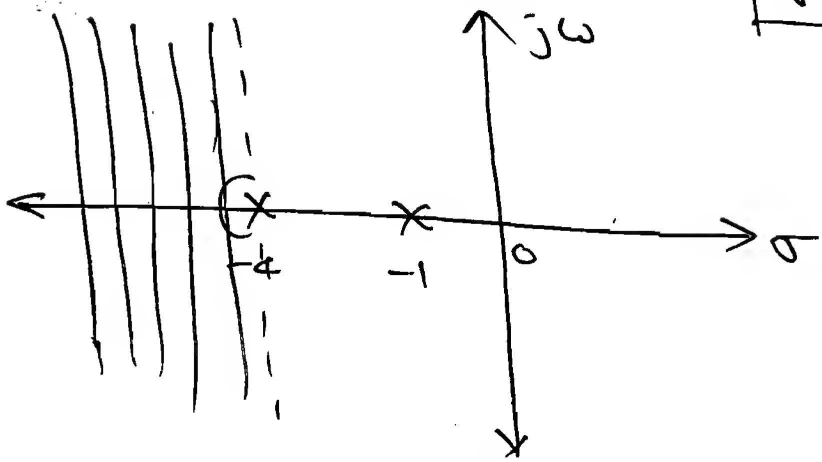
$$= \frac{\frac{1}{3}}{(s+1)} - \frac{1/3}{(s+4)}.$$

① $x(t)$ is right sided, $\boxed{\sigma > -1}$



$\Rightarrow$ $\boxed{x(t) = \frac{1}{3} e^{-t} u(t) - \frac{1}{3} e^{-4t} \cdot u(t).}$

② $x(t)$ is left-sided. $\boxed{\sigma < -4}$



$$\Rightarrow \boxed{x(t) = -\frac{1}{3}e^{-t}u(-t) + \frac{1}{3}e^{-4t}u(-t).}$$

③ $x(t)$ is two sided. $\boxed{-4 < \sigma < -1}$

① $-4 < \sigma$

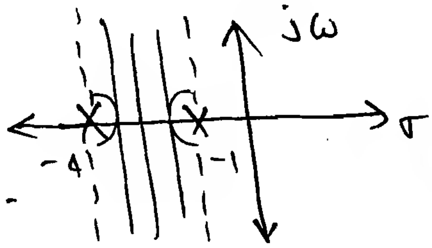
$\Rightarrow \sigma > -4$

$u(t)$

② $\sigma < -1$

$\Downarrow$

$-u(-t)$.



so, $$\boxed{x(t) = -\frac{1}{3}e^{-t}\cdot u(-t) + \frac{1}{3}e^{-4t}\cdot u(t).}$$

$\boxed{\text{P 5.1.1}}$ Given $X(s) = \dfrac{2s+5}{s^2+5s+6}$, find all the time-domain signals?

Soln:

$$X(s) = \frac{2s+5}{s^2+5s+6}$$

$$\Rightarrow X(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{1}{(s+2)} + \frac{1}{(s+3)}$$

① $x(t)$ is left sided.

R.O.C. :- $\boxed{\sigma < -3}$

$$\Rightarrow \boxed{x(t) = -e^{-2t}u(-t) - e^{-3t}u(-t).}$$

② $x(t)$ is Right sided.

R.O.C. $\boxed{\sigma > -2}$

$$\boxed{x(t) = e^{-2t}.u(t) + e^{-3t}.u(t)}$$

③ $x(t)$ is Two sided.

R.O.C. $-3 < \sigma < -2$.

$$\therefore \boxed{x(t) = -e^{-2t}.u(-t) + e^{-3t}.u(t).}$$

2. Time - Shifting:-

$\Rightarrow$ let, $x(t) \longleftrightarrow X(s)$. , Roc = R

then $\boxed{x(t-t_0) \longleftrightarrow e^{-st_0}.X(s)}$, with Roc = R.

P 5.1.6.

(b) $x(t) = u(t-5)$.

Soln:

$$X(s) = e^{-5s}.\ \mathcal{L}[u(t)].$$

$$= e^{-5s}/s.$$

(c) $y(t) = e^{5t} u(-t+3)$.

Sol^n:

$$y(t) = e^{5t} \cdot u(-(t-3))$$

$$\neq \cancel{e^{5t}} \cdot \cancel{e^{-3s}} \; \cancel{L[u(-t)]}$$

$$= e^{-3s} \cdot L[e^{5t} \cdot u(-t)]$$

$$= e^{+3(s-5)} \cdot \frac{1}{-(s-5)}$$

$\boxed{P\,5.1.8}$ Consider the signal $x(t) = e^{-5t} u(t-1)$

with L.T. $X(s)$.

a) Find $X(s)$ with R.O.C?

Sol^n:

$$x(t) = e^{-5t} \cdot u(t-1).$$

$$X(s) = e^{-5t+5-5} \cdot u(t-1).$$

$$= e^{-5(t-1)} \cdot u(t-1) \cdot e^{-5}.$$

$$X(s) = \frac{e^{-5} \cdot e^{-s}}{s+5} \; ; \quad \text{ROC } \boxed{\sigma > -5}$$

(or)

$$X(s) = e^{-s} \left[L\{ e^{-5(t+1)} \right].$$

$$= e^{-5} \left[\frac{e^{-5}}{s+5} \right].$$

$$\Rightarrow \quad X(s) = \frac{e^{-5} \cdot e^{-s}}{s+5} \; ; \quad \text{R.o.C.} \; \boxed{\sigma > -5}$$

(b) Find the values of 'A' & 't_0' such that the L.T. $G(s)$ of $g(t) = A e^{-5t} \cdot u(-t-t_0)$. has same algebraic form as $X(s)$. What is the R.o.C. of corresponding to $G(s)$?

Soln:

$$g(t) = A \cdot e^{-5t} \cdot u(-t-t_0).$$

$$= A \cdot e^{-5t} \cdot u(-(t+t_0)).$$

s became $-s$

$$= A \cdot e^{-t_0 s} \cdot L\left[\, e^{-5(t-t_0)} \,\right].$$

$$= A \cdot e^{5t_0} \cdot e^{-t_0(-s)} \times \frac{1}{-s+5}$$

$$G(s). = \frac{-A \cdot e^{5t_0} \cdot e^{t_0 s}}{s-5} \; ; \quad \text{R.o.C.}$$

$s-5 > 0$

$\sigma > 5$

but $\boxed{\sigma < -5}$

($\because$ s becomes $-s$).

$\therefore$ So, $\boxed{A = -1}$, $\boxed{t_0 = -1}$,

& R.o.c. $\boxed{\sigma < -5}$.

③ Shift in S-domain:-

$\Rightarrow$ If $x(t) \longleftrightarrow X(s)$ with Roc $= R$ then

$$\boxed{x(t) \cdot e^{s_0 t} \longleftrightarrow X(s-s_0) \quad \text{with Roc} = R + \text{Re}(s_0)}$$

(4) Time - reversal :-

$\Rightarrow \quad x(t) \longleftrightarrow X(s)$ then $x(-t) \leftrightarrow X(-s)$,

ROC $= -R$.

$\boxed{P\ 5.1.7.}$ Find the I.L.T. of

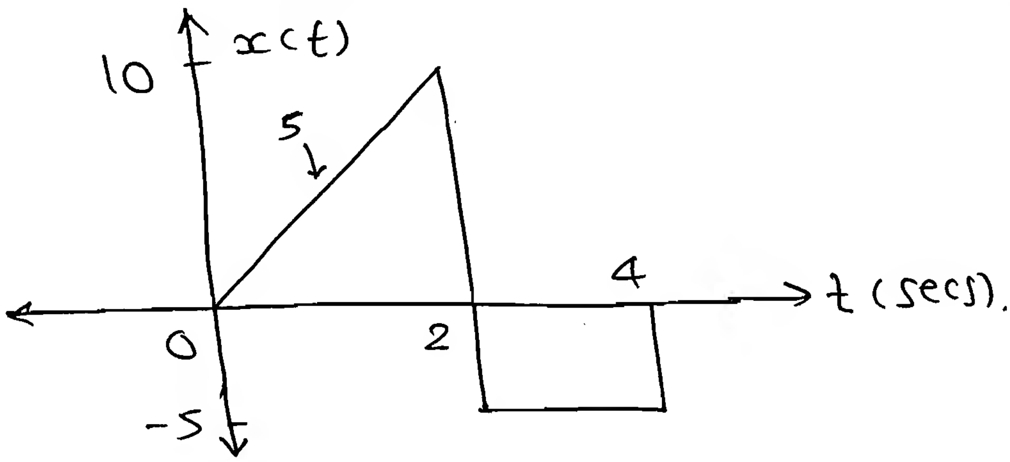
$$Y(s) = \frac{e^{-3s}}{(s+1)(s+2)}, \quad \sigma > -1.$$

$\underline{\underline{Sol^n}}$:

$$Y(s) = \underset{t_0=3\ \downarrow}{e^{-3s}} \left[\frac{1}{(s+1)} - \frac{1}{(s+2)} \right].$$

$$\boxed{y(t) = e^{-(t-3)} \cdot u(t-3) - e^{-2(t-3)} \cdot u(t-3).}$$

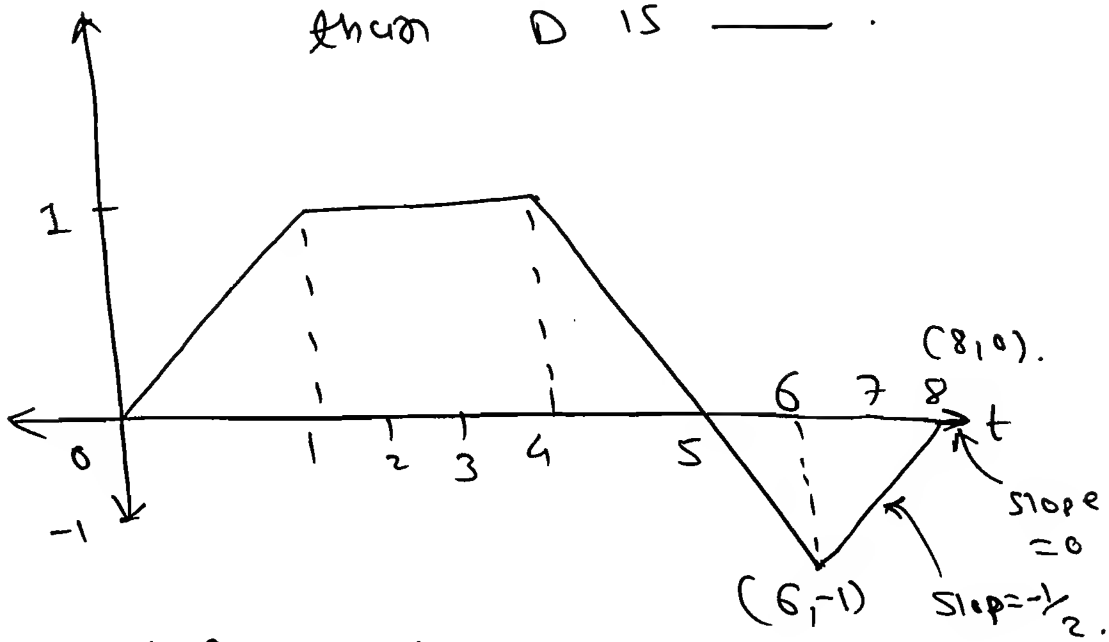
$\boxed{P\ 5.1.9.}$ Find the L.T. of the waveform shown in figure?



$\underline{\underline{Sol^n}}$: $x(t) = 5r(t) - 5r(t-2) - 15u(t-2) + 5u(t-4)$.

$$\Rightarrow X(s) = \frac{5}{s^2} - \frac{5e^{-2s}}{s^2} - \frac{15e^{-2s}}{s} + \frac{5e^{-4s}}{s}.$$

P 5.1.10 L.T. of the waveform shown in is $\frac{1}{s^2}\left(1 + Ae^{-s} + Be^{-4s} + Ce^{-6s} + De^{-8s}\right)$. then D is ____ .



Sol^n:

$$\text{Slop} = \frac{-1-0}{6-8} = \frac{1}{2}.$$

D (8 to 6).

A (1 to 0).

so, $\boxed{D = \frac{1}{2}}$

so, $\boxed{A = \text{Slop} = 1.}$

P 5.1.11 Find the L.T. of

1) $x_1(t) = \cos\omega_0 t\, u(t)$

Sol^n: $x_1(t) = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \cdot u(t).$

$$x_1(t) = \frac{1}{2} \cdot u(t) \cdot e^{j\omega_0 t} + \frac{1}{2} \cdot u(t) \cdot e^{-j\omega_0 t}.$$

let, $x(t) = u(t)$

L.T. $\rightarrow$ $X(s) = 1/s$.

$$X_1(s) = \frac{1}{2}\, X(s - j\omega_0) + \frac{1}{2}\, X(s + j\omega_0).$$

$$\Rightarrow \quad X_1(s) = \frac{1}{2}\left[\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right].$$

$$\therefore \quad \boxed{X_1(s) = \frac{s}{s^2+\omega_0^2}} \quad ; \quad \sigma > 0+0$$

$$\boxed{\sigma > 0}.$$

(2) $x_2(t) = t \cdot e^{-3t}\, u(t).$

Soln: $x_2(t) = t \cdot e^{-3t} \cdot u(t).$

let, $x(t) = t \Rightarrow X(s) = 1/s^2.$

$$\therefore \quad X_2(s) = X(s+3).$$

$$\therefore \quad \boxed{X_2(s) = \frac{1}{(s+3)^2}} \quad ; \quad \boxed{\sigma > 0}$$

(3) $x_3(t) = e^{-at} \cdot \sin\omega_0 t\, u(t).$

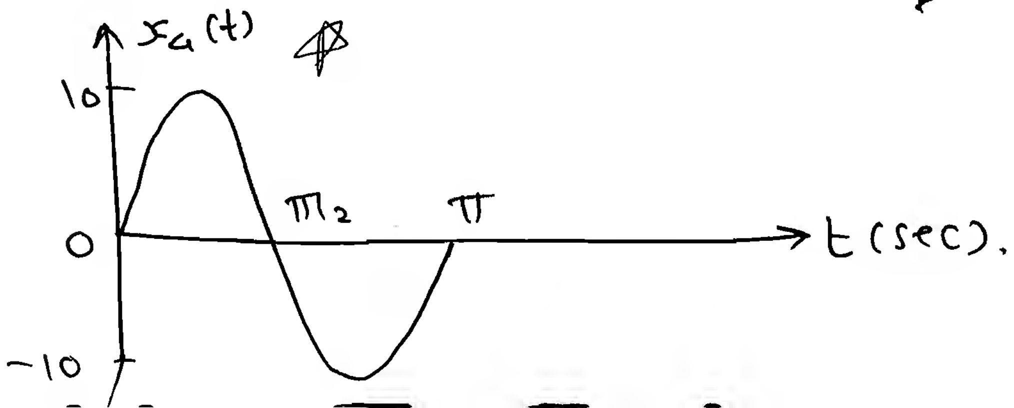
Soln: $L[\sin\omega_0 t] = \dfrac{\omega_0}{s^2+\omega_0^2}.$

$$\therefore \quad X_3(s) = X(s+a).$$

$$\therefore \quad \boxed{X_3(s) = \frac{\omega_0}{(s+a)^2+\omega_0^2}} \quad ; \quad \sigma > 0 + Re\{-a\}$$

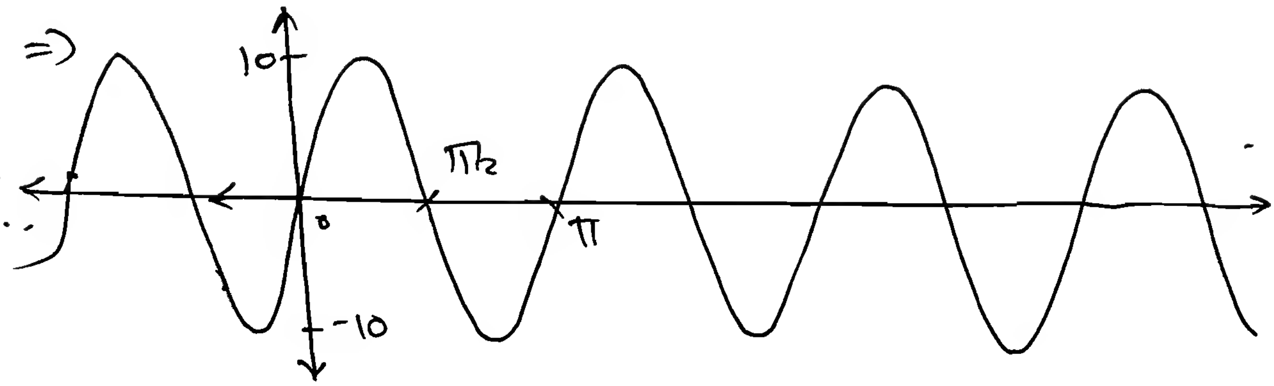
$$; \boxed{\sigma > Re\{-a\}}.$$

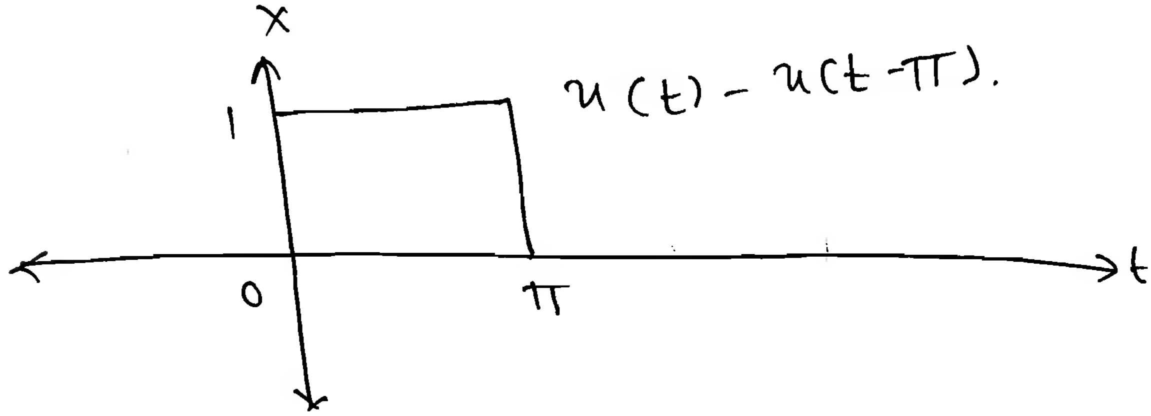
(4)



$$\Rightarrow \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2.$$

$\Rightarrow$





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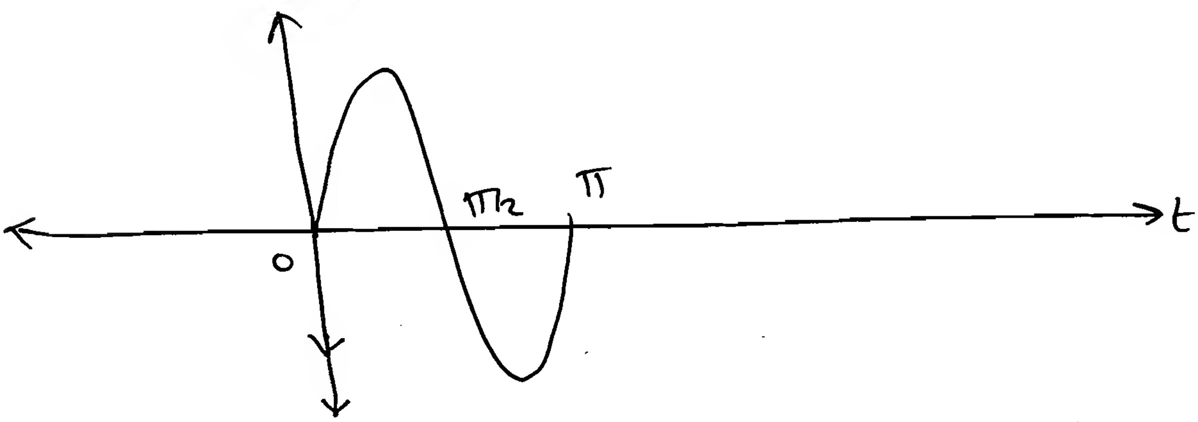


$$\Rightarrow \quad x_a(t) = 10 \sin \omega_0 t \cdot [u(t) - u(t-\pi)].$$

$$x_a(t) = \sin 2t \cdot [u(t) - u(t-\pi)].$$

$$\downarrow^{L.T.} = \frac{20}{s^2+4} - \frac{20e^{-\pi s}}{s^2+4}.$$

$$\boxed{X_a(s) = \frac{20}{s^2+4}\left[1 - e^{-\pi s}\right].}$$

$\boxed{\text{P5.1.12}}$ Let $x(t)$ be a signal that has a rational L.T. with exactly 2 poles located at $s=-1$ and $s=-3$. If $g(t) = e^{2t}x(t)$ & $G(\omega)$ converges, determine whether $g(t)$ is

(a) left sided (b) right-sided.

(c) two - sided (d) finite - duration.

Sol^{n}:

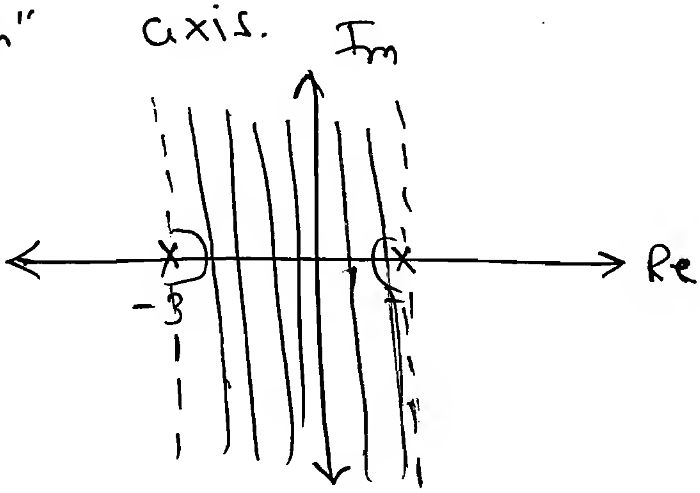
$$X(s) = \frac{1}{(s+1)(s+3)}.$$

$\Rightarrow$ Now, $g(t) = e^{2t} \cdot x(t)$.

$$\Rightarrow \quad G(s) = X(s-2).$$

$$\Rightarrow \quad G(s) = \frac{1}{(s-1)(s+3)}$$

Now, $G(\omega)$ converges means F.T. is define and L.T.'s ROC includes $j\omega$ (or) "Im" axis.



R.o.c. must be

$\Rightarrow$ $\boxed{-3 < \sigma < -1}$ So, Ans (c) two sided

$\boxed{P\ 5.1.13}$ Let $g(t) = x(t) + \alpha x(-t)$ where

$x(t) = \beta e^{-t} u(t)$. & $G(s) = \frac{s}{s^2-1}$, $-1 < Re\{s\} < 1$,

find α & β?

Sol^n:

$$g(t) = x(t) + \alpha x(-t).$$

$$\therefore G(s) = X(s) + \alpha X(-s).$$

$$\Rightarrow x(t) = \beta . e^{-t} u(t).$$

$$\Rightarrow X(s) = \frac{\beta}{(s+1)}, \quad Re\{s\} > -1.$$

$$X(-s) = \frac{\beta}{-s+1}, \quad R\{s\} < 1.$$

$$\therefore G(s) = \frac{\beta}{(s+1)} + \frac{\alpha\beta}{(-s+1)}.$$

$$G(s) = \frac{\beta}{(s+1)} - \frac{\alpha\beta}{s-1}, \quad \text{—— (1)}$$

$$\text{given } G(s) = \frac{s}{s^2-1} = \frac{s}{(s+1)(s-1)}.$$

$$G(s) = \frac{1/2}{(s-1)} - \frac{-1/2}{(s-1)}. \quad \text{—— (2)}.$$

Compare eq^n (1) & (2).

$$\therefore \boxed{\beta = \frac{1}{2}} \qquad \alpha . \beta = -1/2.$$

$$\alpha . \left(\cancel{\tfrac{1}{2}}\right) = \left(-\cancel{\tfrac{1}{2}}\right).$$

$$\boxed{\alpha = -1}$$

Q $Y(s) = \dfrac{s^2 - s + 1}{(s+1)^2}$; $\sigma > -1$.

Solⁿ:

$$Y(s) = \frac{s^2 - s + 1}{(s+1)^2}.$$

$$= \frac{s^2 + 2s + 1 - 3s}{(s+1)^2}.$$

$$= \frac{(s+1)^2 - 3(s+1-1)}{(s+1)^2}.$$

$$Y(s) = 1 - \frac{3}{(s+1)} + \frac{3}{(s+1)^2}.$$

$\downarrow$ I.L.T.

$$\boxed{y(t) = \delta(t) - 3e^{-t}u(t) + 3t \cdot e^{-t} \cdot u(t)}$$

5) Differentiation in time :-

If $x(t) \longleftrightarrow X(s)$ with ROC = R

then $\dfrac{dx(t)}{dt} \longleftrightarrow sX(s)$. with ROC = R.

$\boxed{\text{P5.1.14}}$ Consider 2 right-sided signals $x(t)$ & $y(t)$ related through the equation

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t) \quad \&$$

$$\frac{dy(t)}{dt} = 2x(t).$$

find $X(s)$ & $Y(s)$ with ROCs?

Soln:

$$sX(s) = -2Y(s) + 1$$

& $$sY(s) = 2X(s).$$

$$\Rightarrow Y(s) = \frac{2X(s)}{s}.$$

$$\therefore sX(s) = -2\left[\frac{2X(s)}{s}\right] + 1.$$

$$\therefore sX(s) = -\frac{4X(s)}{s} + 1.$$

$$X(s)\left[s + \frac{4}{s}\right] = 1.$$

$$\therefore \boxed{X(s) = \frac{s}{s^2+4}}. \quad \text{ROC } \boxed{\sigma > 0}$$

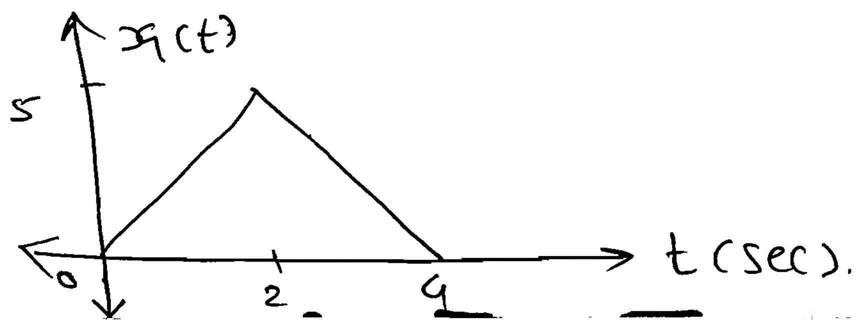
$$\therefore x(t) = \cos 2t$$

$$\therefore Y(s) = \frac{2}{\cancel{s}} \times \frac{\cancel{s}}{s^2+4} = \frac{2}{s^2+4}.$$

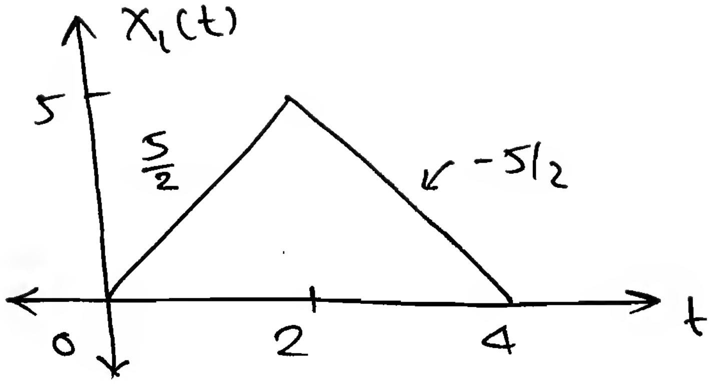
$$\Rightarrow \boxed{Y(s) = \frac{2}{s^2+4}} \Rightarrow y(t) = \sin 2t.$$

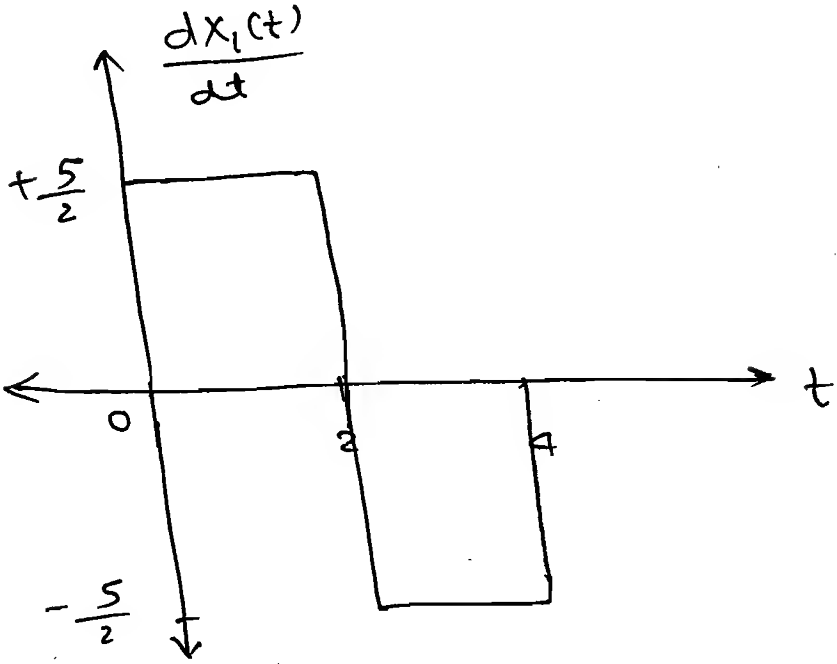
$\sigma > 0$

$\boxed{\text{P 5.1.15}}$ By using derivative method find the L.T. of the following signal?



Sol^n:





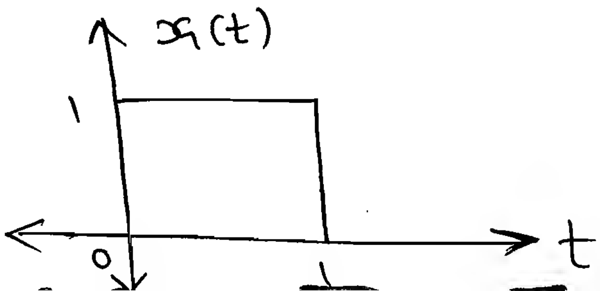
$$\therefore \frac{dx_1(t)}{dt} = \frac{5}{2}u(t) - \frac{10}{2}u(t-2) + \frac{5}{2}u(t-4).$$

$$\therefore sX_1(s) = \frac{5}{2s} - 5.\frac{e^{-2s}}{s} + \frac{5}{2}.\frac{e^{-4s}}{s}.$$

$$\therefore \boxed{X_1(s) = \frac{5}{2s^2} - \frac{5.e^{-2s}}{s^2} + \frac{5.e^{-4s}}{2s^2}.}$$

P 5.1.16 Find the Laplace transform of the following signals?

(1)

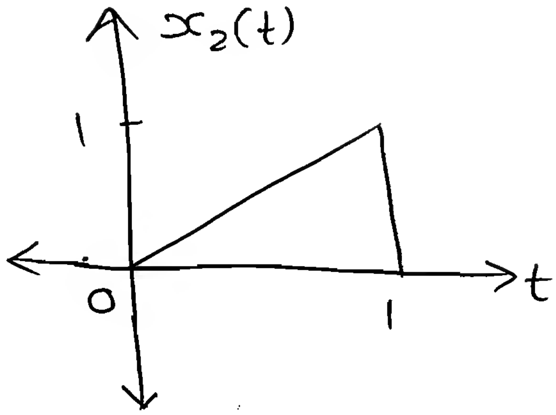


Sol^n: $x_1(t) = u(t) - u(t-1)$.

$\downarrow$ L.T.

$$X_1(s) = \frac{1}{s} - \frac{e^{-s}}{s} = \frac{1-e^{-s}}{s}.$$

(2)



Sol^n: $x_2(t) = t \cdot [u(t) - u(t-1)]$.

$x_2(t) = t \cdot x_1(t)$

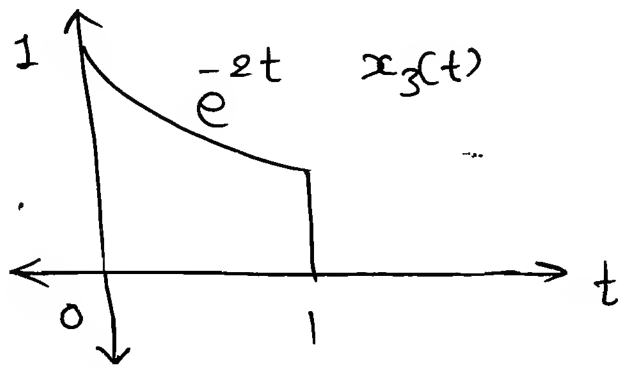
$\downarrow$ L.T.

$$\therefore X_2(s) = (-1)^1 \cdot \frac{d}{ds}(X_1(s)).$$

$$= -\left[\frac{-1}{s^2} - \frac{s(-1)\cdot e^{-s} - e^{-s}}{s^2}\right].$$

$$X_2(s) = \frac{1}{s^2} + \frac{-s e^{-s} - e^{-s}}{s^2}.$$

$\boxed{P\ 5.1.16}$



Sol^n: $x_3(t) = x_1(t)\,[u(t) - u(t-1)]$.

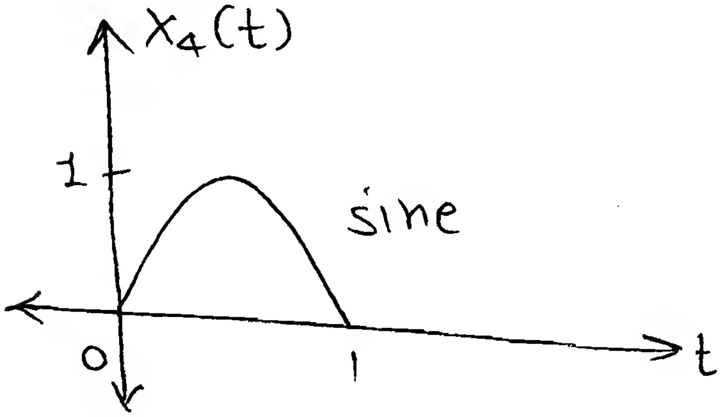
$\therefore \quad x_3(t) = e^{-2t} \cdot x_1(t).$

$$\Rightarrow \quad \downarrow$$

$$X_3(s) = X_1(s+2).$$

$$\boxed{X_3(s) = \frac{1 - e^{-(s+2)}}{(s+2)}}$$

④



$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2}$$

$$\omega_0 = \pi.$$

Sol^n:

$$\therefore \quad x_4(t) = \sin \omega_0 t \cdot [x_1(t)].$$

$$= \sin \pi t \cdot [x_1(t)].$$

$$= \frac{e^{+j\pi t} - e^{-j\pi t}}{2j} \cdot x_1(t).$$

$$x_4(t) = \frac{e^{+j\pi t} \cdot x_1(t) - e^{-j\pi t} \cdot x_1(t)}{2j}$$

$$\Big\downarrow \text{L.T.}$$

$$\therefore \quad X_4(s) = \frac{X_1(s-j\pi) - X_1(s+j\pi)}{2j}.$$

$$\therefore \quad X_4(s) = \frac{1 - e^{-(s-j\pi)}}{(s+2-j\pi)2j} - \frac{1 - e^{-j(s+j\pi+2)}}{(s+j\pi+2)}$$

6) Differentiation in s-domain :-

$\Rightarrow$ If $x(t) \longleftrightarrow X(s)$ with ROC = R,

then $t\,x(t) \longleftrightarrow -\frac{d}{ds} X(s)$ ROC = R.

$$\Rightarrow \boxed{t^n . x(t) \longleftrightarrow (-1)^n . \frac{d^n}{ds^n} X(s).}$$

P 5.1.17 Find the I.L.T. of $X(s) = \log\left[\frac{s+5}{s+6}\right]$?

Soln:

$$X(s) = \log\left[\frac{s+5}{s+6}\right].$$

$$\therefore \frac{dX(s)}{ds} = \frac{1 \cdot \cancel{(s+6)}}{(s+5)} \times \left[\frac{(s+6)(1) - (s+5)}{(s+6)^{\cancel{2}}}\right]$$

$$= \frac{1}{(s+5)(s+6)}.$$

$$\therefore \frac{dX(s)}{ds} = \frac{1}{(s+5)} - \frac{1}{(s+6)}.$$

$$\therefore -t . x(t) = \left(e^{-5t} - e^{-6t}\right) u(t).$$

$$\therefore \boxed{x(t) = \left[\frac{e^{-6t} - e^{-5t}}{t}\right] u(t).}$$

So, in general,

$$\boxed{x(t) = \left[\frac{e^{-\alpha t} - e^{-\beta t}}{t}\right] u(t) \xleftrightarrow{L.T.} \operatorname{Log}\left[\frac{s+\beta}{s+\alpha}\right]}$$

$\boxed{P\,5.1.18}$ Find the ILT of

(a) $$X(s) = \frac{4}{(s+2)(s+1)^3}$$

Soln:

$$X(s) = \frac{4}{(s+2)(s+1)^3}$$

$$X(s) = 4\left[\frac{A=-4}{(s+2)} + \frac{B=4}{(s+1)^3} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)}\right]$$

$$X(s) = \left[\frac{-4}{(s+2)} + \frac{4}{(s+1)^3} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)}\right]$$

$$= \frac{4}{(s+2)(s+1)^3}$$

Put $s=0$.

$$\therefore \frac{4}{2} = -\frac{4}{2} + 4 + C + D.$$

$$\boxed{C=-D}$$

Put $s=1$.

$$\therefore \frac{1}{6} = -\frac{4}{3} + \frac{1}{2} + \frac{C}{4} + \frac{D}{2}.$$

$\Rightarrow$ $C + 2D = 4.$

$\Rightarrow$ $\boxed{D=4}$ & $\boxed{C=-4}$

$$\therefore X(s) = \frac{-4}{(s+2)} + \frac{4}{(s+1)^3} - \frac{4}{(s+1)^2} + \frac{4}{(s+1)}.$$

$\downarrow$ I.L.T.

$$\boxed{x(t) = -4e^{-2t} + \frac{4}{2!}e^{-t}\cdot t^2 - 4te^{-t} + 4e^{-t}}$$

(b) $$X(s) = e^{-2s} \frac{d}{ds}\left[\frac{1}{(s+1)^2}\right].$$

Soln:

$$X(s) = e^{-2s}\left[\frac{-2}{(s+1)^3}\right].$$

$$\downarrow_{I.L.T.} \quad = \frac{-2e^{-2s}}{(s+1)^3}.$$

$$x(t) = -2\frac{(t+2)^2}{2!} \cdot e^{-(t+2)} u(t+2).$$

$$\therefore \boxed{x(t) = -(t+2)^2 \cdot e^{-(t+2)} \cdot u(t+2)}$$

7) Convolution in time :-

⇒ If $x(t) \longleftrightarrow X(s)$ with $Roc = R_1$ &

$h(t) \longleftrightarrow H(s)$ with $Roc = R_2$.

then $\boxed{x(t) * h(t) \longleftrightarrow X(s) \cdot H(s) \quad Roc = R_1 \cap R_2}$

⇒ L.T. of impulse response is known as system (or) transfer function.

$\boxed{P\ 5.1.19}$ Solve the following equation.

$$y(t) + \int_0^\infty y(\tau) . x(t-\tau) d\tau = x(t) + \delta(t) ?$$

Soln: $y(t) + y(t) * x(t) = x(t) + \delta(t)$.

$\xrightarrow{L.T.}$ $Y(s) + \cancel{X(s)} + X(s) \cdot Y(s) = X(s) + 1.$

$\therefore \quad Y(s)\,[\cancel{1 + X(s)}] = \cancel{(1 + X(s))}$

$\therefore \quad y(s) = 1$

$\therefore \quad \boxed{y(t) = \delta(t).}$

P 5.1.20 ~~Solve the following~~

Consider a signal $y(t) = x_1(t-2) * x_2(-t+3)$ where $x_1(t) = e^{-2t} \cdot u(t)$ & $x_2(t) = e^{-3t} \cdot u(t)$. Find $Y(s)$ with ROC?

Soln:

$$X_1(s) = \frac{1}{(s+2)} \quad \sigma > -2, \qquad X_2(s) = \frac{1}{(s+3)} \quad \sigma > -3.$$

$$\therefore \quad y(t) = x_1(t-2) * x_2(-t+3).$$

$$Y(s) = e^{-2s} X_1(s) \cdot e^{-3s} \cdot X_2(-s).$$

$$\therefore \quad \boxed{Y(s) = \frac{e^{-2s}}{(s+2)} \cdot \frac{e^{-3s}}{(-s+3)}}$$

$\sigma > -2$ $\qquad$ $\sigma < 3.$

So ROC $\boxed{-2 < \sigma < 3.}$

P 5.2.21 Find the transfer function & impulse response of a filter whose input-output relation is described by

$$y(t) = x(t) + \int_{-\infty}^{t} y(\lambda) \cdot e^{-3(t-\lambda)} \cdot u(t-\lambda)\, d\lambda.$$

Sol^n:

$$y(t) = \left[x(t) + y(t) * e^{-3t}\right].$$

$$\therefore\ Y(s) = X(s) + Y(s)\cdot\frac{1}{s+3}.$$

$$\therefore\ Y(s)\left[1 - \frac{1}{s+3}\right] = X(s).$$

$$\therefore\ \frac{Y(s)}{X(s)} = H(s) = \frac{s+3}{s+2} = \frac{s+2+1}{(s+2)}.$$

$$\therefore\ H(s) = 1 + \frac{1}{(s+2)}.$$

$$\therefore\ \boxed{h(t) = \delta(t) + e^{-2t}.u(t).}$$

P 5.1.23 An Input $x(t) = \exp(-2t).u(t) + \delta(t-6)$ is applied to an L.T.I. system with impulse response $h(t) = u(t)$. The output is

Sol^n:

$$X(s) = \frac{1}{(s+2)} + e^{-6s}.$$

$$H(s) = \frac{1}{s}.$$

$$\therefore\ H(s) = \frac{Y(s)}{X(s)}.$$

$$\therefore\ Y(s) = H(s)\cdot X(s).$$

$$= \frac{1}{s}\cdot\left[\frac{1}{s+2} + e^{-6s}\right].$$

$$\therefore\ Y(s) = \frac{1}{s(s+2)} + \frac{e^{-6s}}{s}.$$

$$= \frac{1}{2s} - \frac{1}{2(s+2)} + \frac{e^{-6s}}{s}.$$

$$\therefore \quad y(t) = \frac{1}{2}u(t) - \frac{1}{2}e^{-2t}\cdot u(t) + u(t+6).$$

$$\therefore \quad \boxed{y(t) = 0.5\,[1 - \exp(-2)]\,u(t) + u(t+6).}$$

PS.1.14 Let the L.T. of a function $f(t)$ which exists for $t>0$ be $F_1(s)$ and the L.T. of its delayed version $f(t-\gamma)$ be $F_2(s)$. ~~the~~ let $F_1^*(s)$ be the complex conjugate of $F_1(s)$ with $s = \sigma + j\omega$ of

$$G(s) = \frac{F_2(s).\ F_1^*(s)}{|F_1(s)|^2},$$ then the L.T.I. of

$G(s)$ is

(a) $\delta(t)$ (b) $\delta(t-\gamma)$ (c) $u(t)$ (d) $u(t-\gamma)$.

Soln:

$$F_2(s) = e^{-\gamma s}.\ F_1(s).$$

$$\Rightarrow \quad G(s) = \frac{F_2(s).\ F_1^*(s)}{|F_1(s)|^2}.$$

$$\Rightarrow \quad G(s) = \frac{e^{-\tau s}.\ \cancel{F_1(s).\ F_1^*(s)}}{\cancel{|F_1(s)|^2}}$$

$$G(s) = e^{-\tau s}.\ 1$$

$$\Rightarrow \quad \boxed{g(t) = \delta(t-\tau).}$$

Ans: (b) $\delta(t-\gamma)$.

* Frequency Integration :-

$$\Rightarrow \boxed{\frac{x(t)}{t} \longleftrightarrow \int_{s}^{\infty} X(s)\cdot ds.}$$

P 5.1.25 Find the L.T. of $\frac{\sin\omega_0 t}{t}\cdot u(t)$?

Soln: let, $x(t) = \sin\omega_0 t$

$$X(s) = \frac{\omega_0}{s^2+\omega_0^2}$$

$$\Rightarrow L\left[\frac{\sin\omega_0 t}{t}\cdot u(t)\right] = \int_{s}^{\infty} X(s)\cdot ds.$$

$$= \int_{s}^{\infty} \frac{\omega_0}{s^2+\omega_0^2}\cdot ds.$$

$$= \frac{\omega_0}{\omega_0}\tan^{-1}\left(\frac{s}{\omega_0}\right)_s^{\infty}$$

$$= \frac{\pi}{2} - \tan^{-1}(s/\omega_0).$$

* Integration in time :-

$$\Rightarrow \boxed{\int_{0}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{X(s)}{s}.}$$

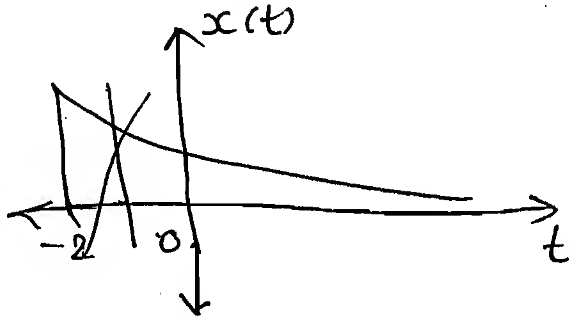
5.2 Unilateral L.T.

$$\boxed{X(s) = \int_{0}^{\infty} x(t)\cdot e^{-st}\cdot dt.}$$

P 5.2.1 Find the U.L.T. of the following signals & find the ROC?

(a) $x(t) = e^{-3t} u(t+2)$.

Soln:



$$\Rightarrow \quad X(s) = \int_{0}^{\infty} e^{-3t} \cdot e^{-st} \cdot dt$$

$$\boxed{X(s) = \frac{1}{s+3} \quad ; \quad \text{R.O.C} \quad \sigma > -3.}$$

(b) $x(t) = \delta(t+2) + \delta(t-4)$.

Soln:

$$\boxed{X(s) = e^{-4s}}$$

* Differentiation in time :-

$$\Rightarrow \quad \frac{d}{dt} x(t) \longleftrightarrow sX(s) - x(0).$$

$$\Rightarrow \quad \frac{d^2}{dt^2} x(t) \longleftrightarrow s^2 X(s) - s x(0) - x'(0).$$

P 5.2.2 A system described by a linear, constant coefficient, ordinary, first order

differential equation has an exact solution given by $y(t)$ for $t>0$, when the forcing function is $x(t)$ and the initial condition is $y(0)$. If one whishes to modify the system so that solution becomes $-2y(t)$ for $t>0$, we need to

Sol: (D) Change the initial condition to $-2y(0)$ and the forcing f^n to $-2x(t)$.

$\boxed{*}$ Total Response

Zero State response (Z.S.R.)
(or)
forced Resp.
(or)
Steady-State Resp.
⇓
I/p is taken.
All initial Conditions are Zero.
⇓
Transfer function

Zero I/p response (Z.I.R.)
(or)
Natural Response.
(or)
Transient Res.
⇓
I/p is not taken.
Initial conditions are considered.
⇓
Charu. Polynomical.

⇒ T.F. of sys. is always representing the Z.S.R.

* Differentiation in time:-

$$\left.\begin{aligned} \frac{d}{dt} &\longrightarrow j\omega \longrightarrow s \\ \frac{d}{d\omega} &\longrightarrow -jt \\ \frac{d}{d(j\omega)} &\longrightarrow -t \\ \frac{d}{ds} &\longrightarrow -t \end{aligned}\right\} ROC = R.$$

* Steady - State Response:-

$\Rightarrow$ If $x(t) = A\cos(\omega_0 t + \theta)$.

then Calculate $H(s)\Big|_{s=j\omega_0}$

$$\Rightarrow \quad H(s)\Big|_{s=j\omega_0} = K\angle\phi.$$

> Steady State response,
>
> $$y_{ss}(t) = KA \cdot \cos(\omega_0 t + \theta + \phi).$$

P 5.2.3 Consider a System with T.F.

$H(s) = \dfrac{s-2}{s^2+4s+4}$. Find the Steady-State response when the input applied is $8\cos 2t$?

Solⁿ: $x(t) = 8\cos 2t.$

$A = 8, \quad \omega_0 = 2, \quad \theta = 0^{\circ}.$

$$H(s)\Big|_{s=2j} = \frac{2j-2}{-\cancel{4}+8j+\cancel{4}} = \frac{j-1}{4j}.$$

$$= \underbrace{0.3536}_{K} \; \underbrace{\angle -45^{\circ}}_{\phi}.$$

$\therefore$ S.S.R. $= y_{ss}(t) = (0.3536 \times 8)\cos(2t + 0^{\circ} + (-45^{\circ}))$

$$\therefore \boxed{y_{ss}(t) = 2\sqrt{2}\cdot\cos(2t - 45^{\circ}).}$$

* Initial & Final Value Theorem :-

$\Rightarrow$ $\boxed{x(0) = \lim_{s \to \infty} sX(s)}$ I.V.T

$\boxed{x(\infty) = \lim_{s \to 0} sX(s).}$ F.V.T.

Note :-

→ Poles at Imaginary axis, F.V.T is not applicable.

→ F.V.T. is valid only if All Poles have −ve Real Parts except a simple Poles have −ve Real Parts except a

simple pole at $s=0$.

$\boxed{P\ 5.2.4.}$ Find the initial & final value for the following T.F.?

a) $X(s) = \dfrac{2s+5}{s^2+5s+6}$.

$\underline{\underline{\text{Sol}^n}}$: I.V.T.

$$x(0) = \lim_{s\to\infty} s.\frac{2s+5}{s^2+5s+6}.$$

$$= \lim_{s\to\infty} \frac{\cancel{s^2}(2+5/s)}{\cancel{s^2}\left(1+\frac{5}{s}+\frac{6}{s^2}\right)}$$

$\therefore$ $\cancel{x(0)=0}$ $\boxed{x(0)=2}$

$\Rightarrow$

F.V.T.

$$x(\infty) = \lim_{s\to 0} \frac{s(2s+5)}{s^2+5s+6}.$$

$$\boxed{x(\infty)=0}$$

(b) $X(s) = \dfrac{4s+5}{2s+1}$.

$\underline{\text{Sol}^n}$:

$\Downarrow$

Proper T.F.

We require strically proper f^n.

$$\Rightarrow X(s) = \frac{4s+2+3}{2s+1}$$

$$X(s) = 2 + \left.\frac{3}{2s+1}\right\} \text{ strically proper.}$$

$$\Rightarrow \quad x(0) = \lim_{s\to\infty} s\left[\frac{3}{2s+1}\right]$$

$$= \lim_{s\to\infty} \frac{3}{2+1/s}$$

$$\boxed{x(0) = 3/2}$$

$$\Rightarrow \quad x(\infty) = \lim_{s\to 0} s\left[\frac{3}{2s+1}\right]$$

$$\therefore \boxed{x(\infty) = 0}$$

(c) $X(s) = \dfrac{12(s+2)}{s(s^2+4)}$

$\underline{\underline{\text{Sol}^n}}$, $\quad x(0) = \lim_{s\to\infty} \dfrac{\cancel{s}\cdot 12(s+2)}{\cancel{s}(s^2+1)} = \dfrac{12(1+\frac{2}{s})}{s(1+\frac{4}{s^2})}$

$= 0.$

here, Pole is on imaginary axis.

so, final value can not be determine.

(d) $X(s) = e^{-s}\left[\dfrac{-2}{s(s+2)}\right]$

$\underline{\underline{\text{Sol}^n}}$: $\quad x(0) = \lim_{s\to\infty} \cancel{s}\cdot e^{-s}\left[\dfrac{-2}{\cancel{s}(s+2)}\right]$

$$\therefore \boxed{x(0) = 0}$$

$$\Rightarrow \quad x(\infty) = \lim_{s\to 0} \cancel{s}\cdot e^{-s}\left[\frac{-2}{\cancel{s}(s+2)}\right]$$

$$= \frac{-2}{2}$$

$$\boxed{x(\infty) = -1.}$$

$\boxed{a}$ *** $X(s) = \dfrac{s}{s^2+\omega_0^2}$.

Soln:

$x(\infty) = \lim_{s\to 0} \dfrac{s^2}{s^2+\omega_0^2} = 0 . X$

I.L.T.

but

$x(t) = \cos \omega_0 t \cdot u(t)$

$$\boxed{-1 \le x(t)\big|_{t\to\infty} \le 1}$$

***

$\boxed{a}$ $X(s) = \dfrac{1}{s-2}$,

Soln:

$x(\infty) = \lim_{s\to 0} \dfrac{s}{s-2} = 0$ X

I.L.T.

$x(t) = e^{2t}$

$x(t)\big|_{t\to\infty} = \infty$

$\boxed{P\,5.2.5}$ A LTI, Causal continuous time System has a rational T.F with simple Poles at $s=-2$, and $s=-4$ and one of the simple ~~Pole~~ zero at $s=-1$. A unit Step $u(t)$ is applied as the input of the system. At steady state, the output has a constant value of 1. Find the impulse response?

Soln:

$$H(s) = \frac{K(s+1)}{(s+2)(s+4)}.$$

$$\Rightarrow \quad y(\infty) = \lim_{s \to 0} s \cdot Y(s) = 1.$$

$$\therefore \quad \lim_{s \to 0} s \cdot H(s) \cdot X(s) = 1.$$

$$\Rightarrow \quad \lim_{s \to 0} \cancel{s} \cdot \frac{K(s+1)}{(s+2)(s+4)} \cdot \frac{1}{\cancel{s}} = 1$$

$$\Rightarrow \quad \frac{K(0+1)}{(0+2)(0+4)} = 1$$

$$\boxed{K = 8}$$

$$\Rightarrow \quad H(s) = \frac{8(s+1)}{(s+2)(s+4)}.$$

$$\Rightarrow \quad H(s) = \frac{12}{(s+4)} - \frac{4}{(s+2)}.$$

$$\Rightarrow \quad \boxed{h(t) = 12e^{-4t} \cdot u(t) - 4e^{-2t} \cdot u(t).}$$

P 5.2.6. An LTI system having TF $\frac{s^2+1}{s^2+2s+1}$ & input $x(t) = \sin(t+1)$ is in steady state. The output is sampled at ω_s rad/sec to obtain final output $\{y(k)\}$ which of the following is true?

(a) $y(\cdot) = 0$ for all ω_s.

(b) $y(\cdot) \neq 0$ for all ω_s.

(C) $y(\bullet) \neq 0$ for $\omega_s > 2$ but zero for $\omega_s < 2$.

(D) $y(\bullet) = 0$ for $\omega_s > 2$ but nonzero for $\omega_s < 2$.

Sol^n^:

$$H(s) = \frac{s^2+1}{s^2+2s+1}.$$

$$H(j\omega) = \frac{-\omega^2+1}{-\omega^2+2j\omega+1}.$$

$$H(j\omega)\Big|_{\omega=1} = \frac{-1+1}{-1+2j+1} = 0.$$

So, ans: (A) $y(\bullet) = 0$ for all ω_s.

P 5.2.7 What is the output as $t \to \infty$ for a system that has T.F., $G(s) = \frac{2}{s^2-s-2}$ when subjected to a step input?

(a) -1 (B) 1 (C) 2 (D) unbounded.

Sol^n^:

$$G(s) = \frac{2}{s^2-s-2}.$$

$$= \frac{2}{s^2-s+\frac{1}{4}-\frac{9}{4}}.$$

$$G(s) = \frac{2}{(s-\frac{1}{2})^2-(3/2)^2}.$$

Poles: $s_1, s_2 : \frac{1}{2} \pm 3/2 = 2, -1.$

Poles are Right hand side of the

s plane. So, Ans: (D) unbounded.

$\boxed{P\ 5.2.8}$ Consider a system described by the T.F. $G(s) = \frac{2s+3}{s^2+2s+5}$ when it is subjected to an input of $10u(t)$, find the initial & final values of the response.

Soln:

$$Y(s) = H(s).\ X(s).$$

$$Y(s) = \frac{2s+3}{s^2+2s+5} \times \frac{10}{s}.$$

$$\therefore\ y(0) = \lim_{s\to\infty} \cancel{s} \times \frac{10}{\cancel{s}} \cdot \times \frac{2s+3}{(s^2+2s+5)}$$

$$\boxed{y(0) = 0}$$

$$\Rightarrow\ y(\infty) = \lim_{s\to 0} \cancel{s} \times \frac{10}{\cancel{s}} \times \frac{2s+3}{(s^2+2s+5)}$$

$$= \frac{\overset{2}{\cancel{10}} \times 3}{(0+0+\cancel{5})}$$

$$\boxed{y(\infty) = 6}$$

$\boxed{P\ 5.2.9}$ Let a signal $a_1 \sin(\omega_1 t + \phi_1)$ be applied to a stable LTI system. Let the corresponding steady state output be represented as $F_2(\omega_2 t + \phi_2)$. Then which of the following statement is TRUE?

(A) F is not neccessarily a "sine" or "Cosine" function but must be periodic & $\omega_1 = \omega_2$

(B) F must be "sine" (or) "Cosine" with $a_1 = a_2$

(C) F must be "sine", $\omega_1 = \omega_2$, $a_1 \neq a_2$.

(D) F must be "sine" (or) "Cosine" functions with $\omega_1 = \omega_2$.

Soln: Input freq. and output freq. Should remain same. so. $\omega_1 = \omega_2$.

→ Function can be sine (or) cosine.

→ Amplitude can be change.

so, Ans - (D).

* Causality & Stability :-

⇒ For a causal system $h(t) = 0$; $t < 0$ and thus is right - sided ang the Roc associated with the system fn for a causal system is a right-half plane.

⇒ An LTI system is stable if and only if the Roc of the system function $H(s)$ include $j\omega$ axis.

$\Rightarrow$ For the T.F. to be both causal and stable all poles must lie in the left half of s-plane with -ve Real parts.

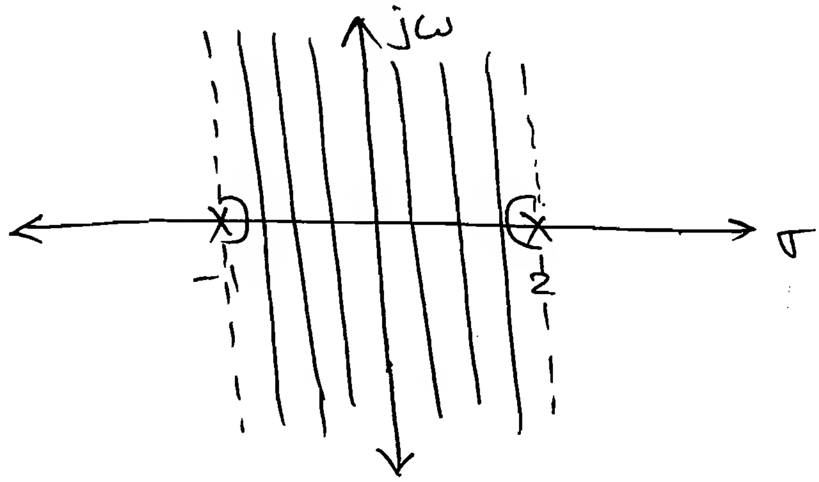
$\boxed{P\ 5\ 3.1}$ Given $H(s) = \dfrac{s-1}{(s+1)(s\pm2)}$. Find $h(t)$ for each of the following cases.

(i) Stable.

Soln:

$$H(s) = \frac{s-1}{(s+1)(s-2)}.$$

$$\Rightarrow H(s) = \frac{\frac{2}{3}}{(s+1)} + \frac{\frac{1}{3}}{(s-2)}.$$



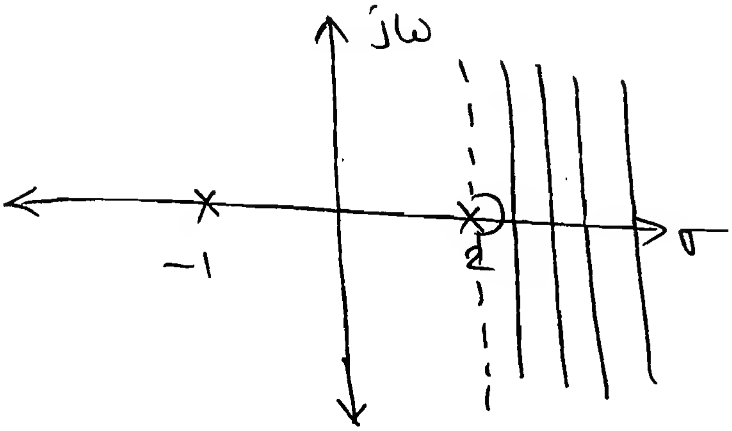
$$\Rightarrow \boxed{-1 < \sigma < 2}$$

$$\therefore \boxed{h(t) = \frac{2}{3}e^{-t}u(t) - \frac{1}{3}e^{2t}\cdot u(-t).}$$

(ii) Causal:

$$\Rightarrow H(s) = \frac{\frac{2}{3}}{(s+1)} + \frac{\frac{1}{3}}{s-2}.$$

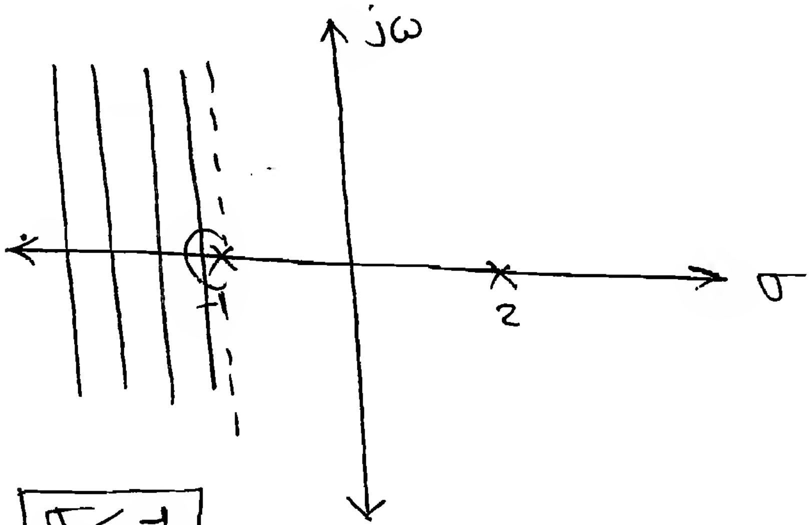
$\Rightarrow$



$$\boxed{\sigma > 2}$$

So, $$\boxed{h(t) = \frac{2}{3} \cdot e^{-t} u(t) + \frac{1}{3} e^{2t} \cdot u(t).}$$

(iii) neither causal nor stable.



$$\boxed{\sigma < -1}$$

So, $$\boxed{h(t) = -\frac{2}{3} e^{-t} u(-t) + \left(-\frac{1}{3}\right) \cdot e^{2t} \cdot u(-t).}$$

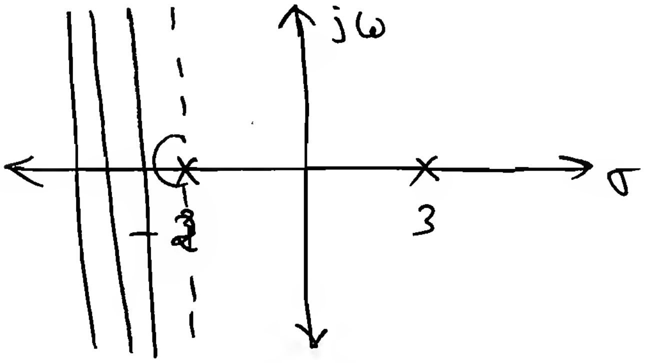
$\boxed{\text{PS. 3.2}}$ Given $X(s) = \dfrac{5-s}{s^2-s-6}$ & F.T. of the signal is defined then $x(t)$ is ______.

Soln: The given system is unstable.

$$X(s) = \frac{5-s}{s^2-s-6}.$$

$$X(s) = \frac{5-s}{(s-3)(s+2)}.$$

$$\Rightarrow X(s) = \frac{\frac{2}{5}}{(s-3)} - \frac{7/5}{(s+2)}.$$



$\boxed{\sigma < -2}$

$$\therefore \boxed{x(t) = -\frac{2}{5} e^{3t} u(-t) + \frac{7}{5} e^{-2t} \cdot u(-t).}$$

$\boxed{P\ 5.3.3}$ Consider an LTI system for which we are given the following information $X(s) = \frac{s+2}{s-2}$ and $x(t) = 0,\ t > 0$, and output is $y(t) = -\frac{2}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(t)$.

(a) Find T.F. & R.o.C.?

(b) Find the output if input is $x(t) = e^{3t}$ using part (a)?

$\underline{\text{Sol}^n}$: $X(s) = \frac{s+2}{s-2}$ and $\underline{x(t) = 0,\ t > 0}$

i.e. signal is left sided.

Hence, R.o.C $\boxed{\sigma < 2}$

Now, $y(t) = -\frac{2}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(t).$

$$Y(s) = \frac{\frac{2}{3}}{(s-2)} + \frac{\frac{1}{3}}{(s+1)}$$

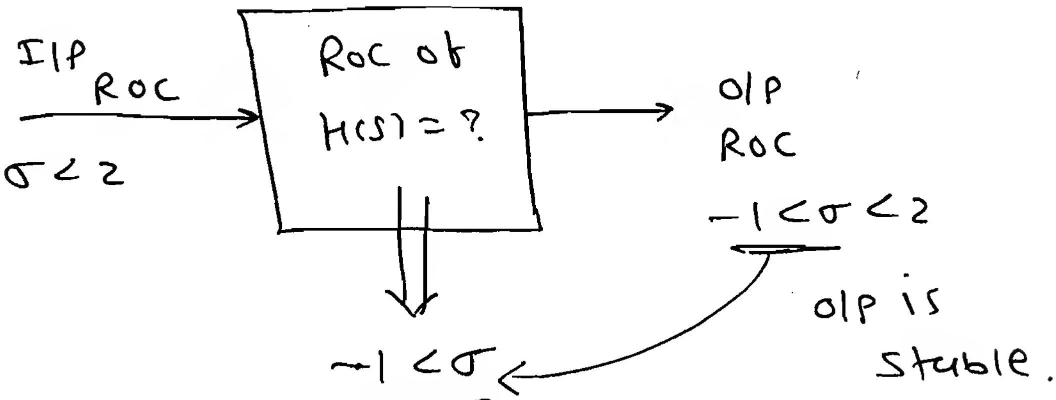
$$= \frac{1}{3}\left[\frac{2s+\cancel{2} + s-\cancel{2}}{(s-2)(s+1)}\right]$$

$$Y(s) = \frac{s}{\underset{\sigma<2}{(s-2)}\,\underset{\sigma>-1}{(s+1)}} \Rightarrow \boxed{-1<\sigma<2}$$

$$\text{T.F.} \Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{s}{(s-2)(s+1)}}{\frac{(s+2)}{(s-2)}}$$

$$= \frac{s}{\cancel{(s-2)}(s+1)} \times \frac{\cancel{s-2}}{(s+2)}$$

$$H(s) = \frac{s}{(s+1)(s+2)}$$



$\therefore \boxed{\sigma>-1}$

$$\therefore \boxed{H(s) = \frac{s}{(s+1)(s+2)} \; ; \; \text{ROC}: \sigma>-1}$$

(b) ~~Find~~ ~~f~~

$x(t) = e^{3t}$

if i/p $x(t) = e^{st} \Rightarrow$ o/p $y(t) = e^{st} \cdot H(s)$.

here, $\boxed{s=3}$

$$\therefore \quad y(t) = e^{3t} \cdot H(s)\Big|_{s=3}$$

$$= \frac{e^{3t} \times 3}{(4)(5)}.$$

$$\boxed{y(t) = \frac{3}{20} \cdot e^{3t}}$$

$\boxed{\text{P 5.3.4}}$ Consider an LTI system with input $x(t)$ and output $y(t)$ related as

$$\frac{dy(t)}{dt} + 3y(t) = \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} - 2x(t).$$

Find the T.F. of inverse system. Does a stable & causal inverse system exist?

<u>Soln</u>:

$$\xrightarrow{\text{L.T.}} \quad sY(s) + 3Y(s) = s^2X(s) + sX(s) - 2X(s).$$

$$\therefore \quad \frac{Y(s)}{X(s)} = H(s) = \frac{(s^2+s-2)}{(s+3)}.$$

$$\therefore \quad H_{Inv}(s) = \frac{(s+3)}{(s^2+s-2)}.$$

$$H_{Inv}(s) = \frac{(s+3)}{(s+2)(s-1)}.$$

for causal system poles must be lies on left hand side so can't be causal & stable simultaneously.

$\Rightarrow$ The system is stable if $\boxed{-2 < \sigma < 1}$.

P5.3.5 Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI sys?

(A) All the poles of the system must lie on the left side of the $j\omega$ axis.

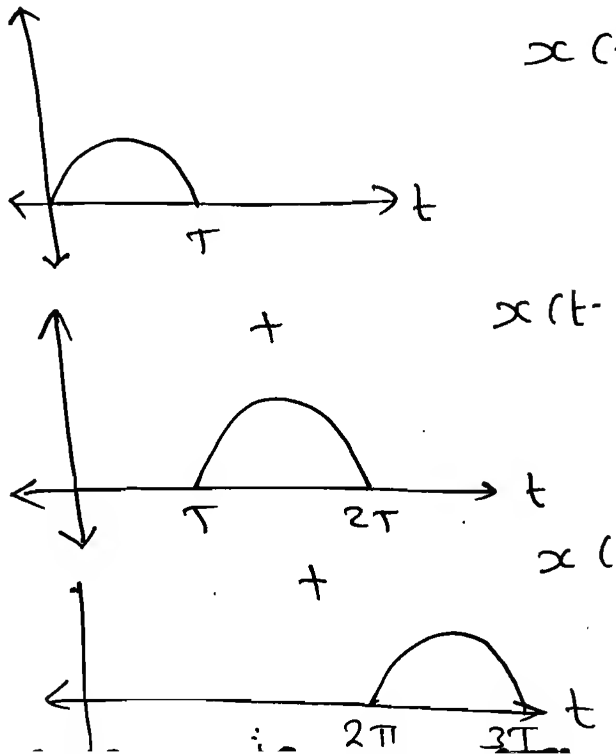
(B) Zeros of the system can lie anywhere in the s-plane.

(C) All the poles must lie within $|s| = 1$.

(D) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis.

Ans: (C)

* L.T. of Switched Periodic Signal:-



$\Rightarrow$

$x(t) \cdot u(t) \leftrightarrow X(s)$

$+$

$x(t-T) \cdot u(t-T) \leftrightarrow e^{-sT} \cdot X(s).$

$+$

$x(t-2T) \cdot u(t-2T) \leftrightarrow e^{-2sT} \cdot X(s).$

$+$

$\vdots$

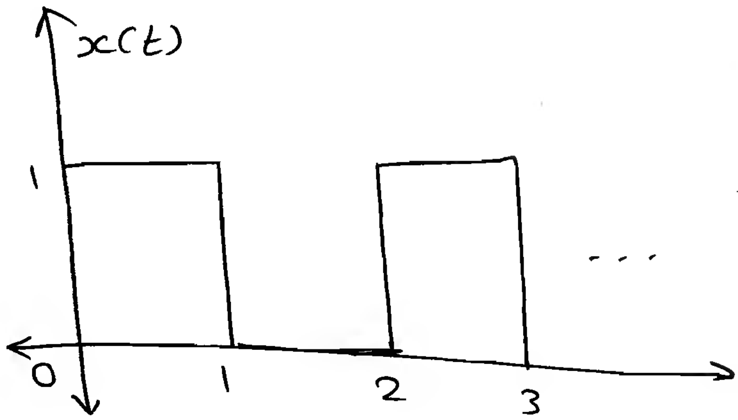
$$\Rightarrow Y(s) = X(s)\left[1 + e^{-sT} + e^{-2sT} + e^{-3sT} + \cdots\right].$$

$$\therefore Y(s) = X(s) \times \frac{1}{1-e^{-sT}}.$$

$$Y(s) = \frac{X(s)}{1-e^{-sT}}.$$

$$\Rightarrow \boxed{Y_p(s) = \frac{\int_0^T x(t).e^{-sT}.dt}{1-e^{-sT}}}$$

e.g Find the L.T. of the periodic signal shown in fig.2.



Soln: $T = 2$

$$X_p(s) = \frac{\int_0^T x(t).e^{-st}.dt}{1-e^{-sT}}.$$

$$= \frac{\int_0^2 x(t).e^{-st}dt}{1-e^{-2s}}.$$

$$= \frac{\int_0^1 1.e^{-st}.dt}{1-e^{-2s}}.$$

$$= \frac{\left[\frac{e^{-st}}{-s}\right]_0^1}{1-e^{-2s}}$$

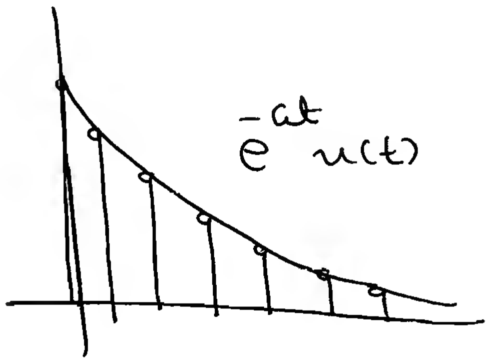
$$= \frac{\frac{1}{s} - \frac{e^{-s}}{s}}{1-e^{-2s}}.$$

$$\therefore \boxed{X_p(s) = \frac{1-e^{-s}}{s(1-e^{-2s})}}$$

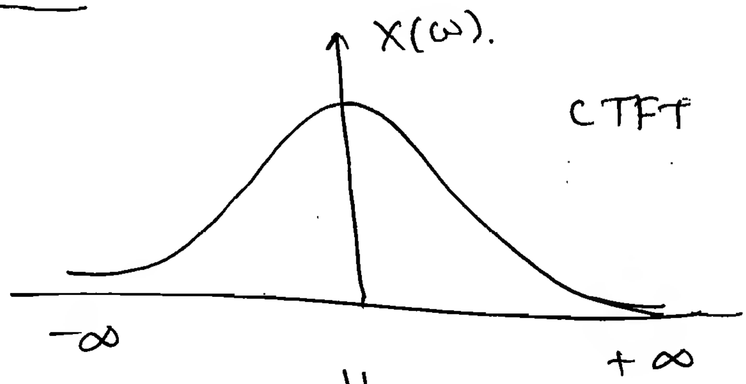
Ch- 6 - DTFT

- The DTFT describes the spectrum of discrete signals & formalizes that discrete signals have periodic spectra.

→ The freq. range for a discrete signal is unique over $(-\pi, +\pi)$ (or) $(0, 2\pi)$.



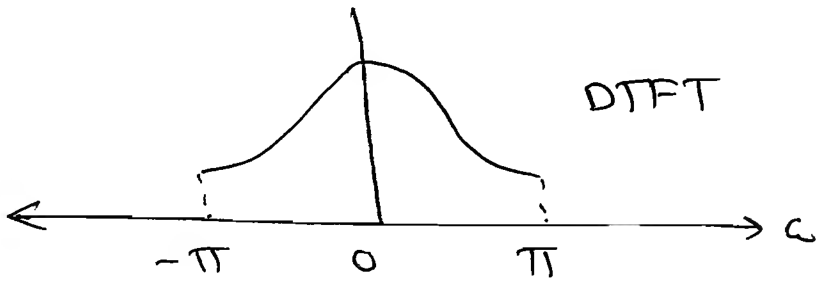
$$e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}.$$



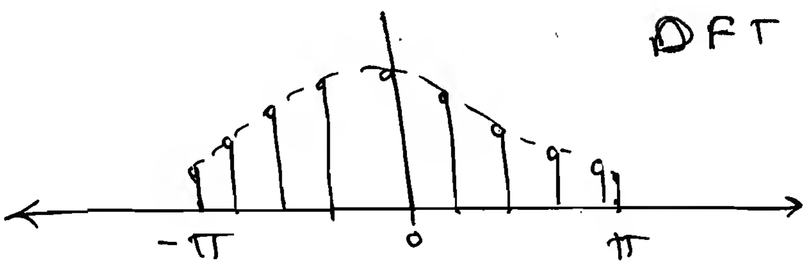
⇓

$x[n] \longleftrightarrow X(e^{j\omega})$

(or) $X(\omega)$.



⇓



⇒ CT FT	D.T. F.T.
ω: $-\infty$ to $+\infty$	ω : $-\pi$ to $+\pi$ (or) 0 to 2π
non-periodic	periodic

⇒ D.T.F.T

$$\boxed{X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n).\, e^{-j\omega n}}$$

⇒ I.DT.F.T.

$$\boxed{x(n) = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(e^{j\omega}).\, e^{j\omega n}\, d\omega.}$$

⇒ $\boxed{\text{F.S.} \xleftrightarrow{\text{Dual}} \text{D.T.F.T}}$

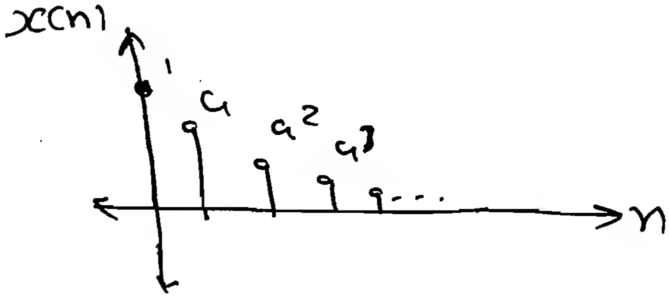
* Convergence of DTFT.

⇒ A sufficient condition for existance of DTFT is $\boxed{\sum_{n=-\infty}^{+\infty} |x(n)| < \infty.}$

- Some sequences are not absolutely summable, but they are square summable.
- There are signals that are neither absolutely summable nor have finite energy, but still have DTFT.

e.g. $x[n] = a^n . u[n] \; ; \quad |a|<1.$

$\Rightarrow$



$$\therefore \; X_1(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) . e^{-j\omega n}$$

$$= \sum_{n=+\infty}^{+\infty} a^n . e^{-j\omega n}$$

$$= \sum_{n=0}^{+\infty} (a . e^{-j\omega})^n$$

$$= 1 + a e^{-j\omega} + (a e^{-j\omega})^2 + \dots$$

$$\therefore \; \boxed{X_1(e^{j\omega}) = \frac{1}{1 - a e^{-j\omega}}}.$$

$$\Rightarrow \; X_1(e^{j\omega}) = \frac{1}{1 - a(\cos\omega - j\sin\omega}.$$

$$X_1(e^{j\omega}) = \frac{1}{(1 - a\cos\omega) + ja\sin\omega}.$$

$$|X(\omega)| = \frac{1}{\sqrt{(1-a\cos\omega)^2 + a^2\sin^2\omega}}.$$

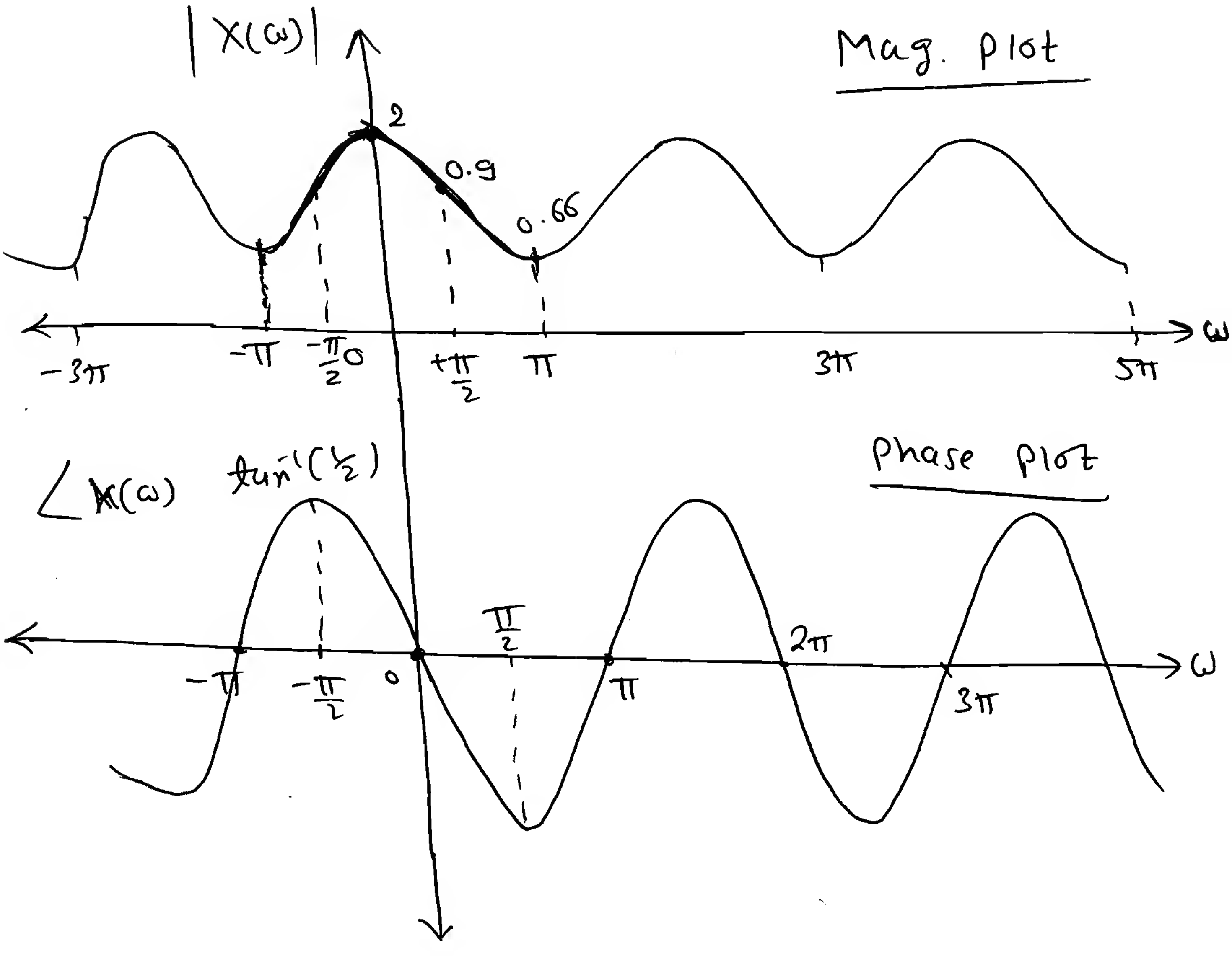
$$= \frac{1}{\sqrt{1 - 2a\cos\omega + a^2}}$$

let, $a = \frac{1}{2}$.

$$\therefore \; |X(\omega)| = \frac{1}{\sqrt{1.25 - \cos\omega}}$$

$$\angle X(\omega) = -\tan^{-1}\left[\frac{a\sin\omega}{1 - a\cos\omega}\right].$$

$$\angle X(\omega). = -\tan^{-1}\left[\frac{\frac{1}{2}\sin\omega}{1 - \frac{1}{2}\cos\omega}\right].$$



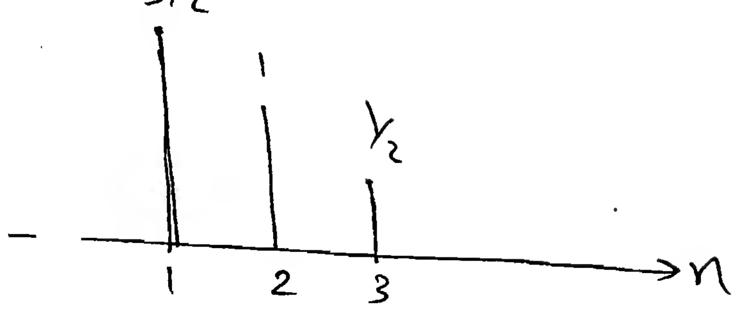
* Periodicity Property:

$$\Rightarrow \boxed{X\left(e^{j(\omega+2\pi)}\right) = X\left(e^{j\omega}\right).}$$

...er or not the n F.T. of the signal

frequency response ?

$x_1(n)$



N_1

$$\frac{\sin\left(\omega\left(3+\frac{1}{2}\right)\right)}{\sin(\omega/2)}$$

$$\sum_{-\infty}^{\infty} x[n]\cdot e^{-j\omega n}$$

Causal.

$$\sum_{\infty}^{\infty} x(n)\cdot e^{-j\omega n}$$

$$+ e^{j\omega 3}\Big] + 1\Big[e^{-j\omega\cdot 2} + e^{j\omega 2}\Big]$$

then

$\boxed{(e^{j\omega})}$

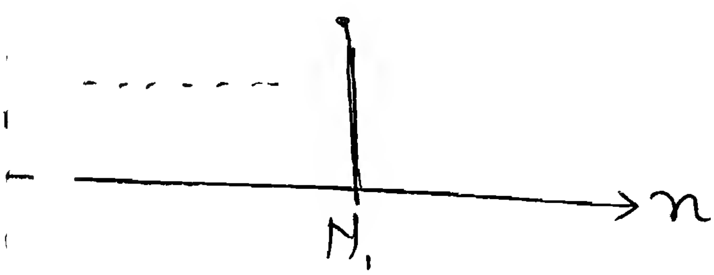
$$\frac{3}{2}\Big[e^{-j\omega} + e^{j\omega}\Big] + 2.$$

$$\boxed{+ \cancel{\tfrac{1}{2}}2\cos 2\omega + 3\cos\omega + 2}$$

then

$\boxed{e^{j(\omega-\omega_0)}}$

$..e^{-j\omega}$.



Sol^n:

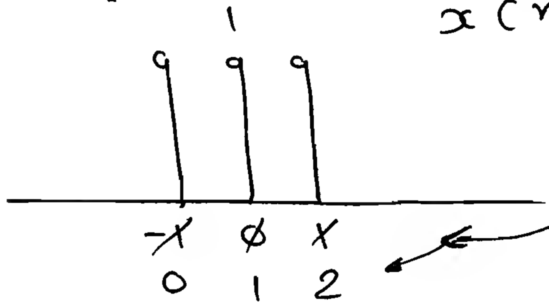
$$H(\omega) = \frac{\sin\left(\omega\left(\overset{N_1 = 1}{1} + \frac{1}{2}\right)\right)}{\sin(\omega/2)} \cdot \cancel{e^{+j\omega(-1)}} \cdot e^{-j\omega(1)}$$

$N_0 = 1$

$\downarrow$ I.D.T.F.T

$\overset{N_0 = 1}{x(n-1)}.$

$\therefore$



So, given system is Causal.

(c) $H(\omega) = e^{-j3\omega} \cdot 1 + e^{+j2\omega} \cdot 1.$

Sol^n:

$$\delta(n) \xleftrightarrow{\text{D.T.F.T}} 1.$$

$$\therefore \quad h(n) = \delta(n-3) + \delta(n+2).$$

$\Downarrow$

given system is Non-Causal.

$\boxed{\text{P 6.1.2.}}$ (a) Let $x(n) = \left(\frac{1}{2}\right)^n \cdot u(n)$, $y(n) = x^2(n)$. & $Y(e^{j\omega})$ be the F.T. of $y(n)$. Then $Y(e^{j0})$ is ______

Sol^n:

$$Y(\omega) = \sum_{n=-\infty}^{\infty} y(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{+\infty} x^2(n) \cdot e^{-j\omega n}.$$

$\Rightarrow \quad x(n) = \left(\frac{1}{2}\right)^n . u(n).$

$x^2(n) = \left(\frac{1}{2}\right)^{2n} . \; u(n) = \left(\frac{1}{4}\right)^n . u(n).$

$\therefore \; Y(\omega) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n . \; e^{-j\omega n}.$

$\therefore \; Y(e^{j0}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n . \; e^{-j\omega(0)}.$

$\therefore \; Y(e^{j0}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n.$

$\Rightarrow \; y(e^{j0}) = \frac{1}{1-1/4} = \frac{4}{3}.$

$\therefore \; \boxed{Y(e^{j0}) = \frac{4}{3}.}$

(b) Given $X(e^{j\omega}) = \cos^3(3\omega)$, the find the sum $S = \sum_{n=-\infty}^{+\infty} (-1)^n . X(n).$

Sol^n:

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) . e^{-j\omega n}.$

let, $\omega = \pi$

$\therefore \; X(e^{j\pi}) = \sum_{n=-\infty}^{\infty} x(n) . e^{-j\pi n}$

$\therefore \; X(e^{j\pi}) = \sum_{n=-\infty}^{+\infty} x(n) . (-1)^n = S.$

$$\therefore \quad S = \sum_{n=-\infty}^{\infty} (-1)^n \cdot x(n) = X(e^{j\pi})$$

$$= \cos^3(3\pi)$$

$$\therefore \quad \boxed{S=-1} \qquad = (-1)^3 = -1.$$

(c) What is the d.c. & high-frequency gain of the filter described by $h(n) = \{1, 2, 3, 4\}$.

Soln:

$$H(\omega) = \sum_{n=-\infty}^{+\infty} h(n) \cdot e^{-j\omega n}.$$

$$\therefore \quad H(\omega) = \sum_{n=0}^{3} h(n) \cdot e^{-j\omega n}.$$

$$\therefore \quad H(\omega) = 1 + 2e^{-j\omega(1)} + 3e^{-j2\omega} + 4e^{-j3\omega}.$$

for d.c. gain $\omega = 0$.

$$\therefore \quad H(0) = \sum_{n=0}^{3} h(n) \cdot (1). = 1+2+3+4 = 10.$$

for high freq. gain $\omega = \pi$

$$\therefore \quad H(e^{j\pi}) = \sum_{n=0}^{3} h(n) \cdot (-1)^n. = 1-2+3-4 = -2.$$

$$\boxed{\begin{array}{l} \text{D.C. gain} = H(0) = \displaystyle\sum_{n=-\infty}^{\infty} x(n). \\ \text{H.F. gain} = H(e^{j\pi}) = \displaystyle\sum_{n=-\infty}^{+\infty} x(n) \cdot (-1)^n \end{array}}$$

P 6.1.3. Find the F.T. of

(i) $y_1(n) = \left(\frac{1}{4}\right)^n \cdot u(n-3).$

Soln:

$$y_1(n) = \left(\frac{1}{4}\right)^{(n-3)+3} \cdot u(n-3).$$

$\downarrow$ F.T

$$= \frac{1}{64} \cdot \left[\left(\frac{1}{4}\right)^{n-3} \cdot u(n-3) \right]. \quad n_0 = 3$$

$$\boxed{Y(\omega) = \frac{1}{64} \cdot \frac{e^{-j3\omega}}{1 - \frac{1}{4} \cdot e^{-j\omega}}}$$

(ii) $y_2(n) = \delta(n-n_0).$

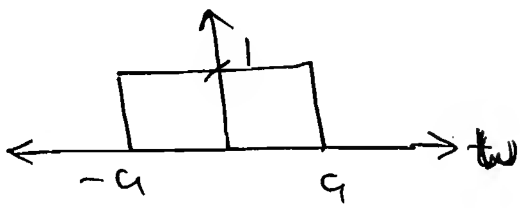
Soln:

$$y_2(n) = \delta(n-n_0).$$

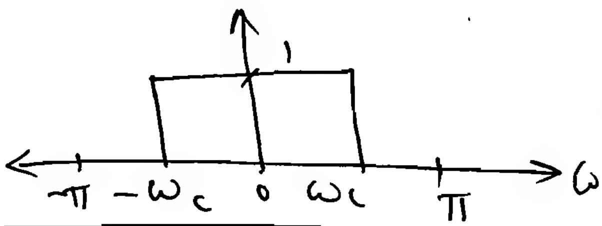
$\downarrow$ F.T

$$\boxed{Y(\omega) = e^{-j\omega n_0} \cdot 1.}$$

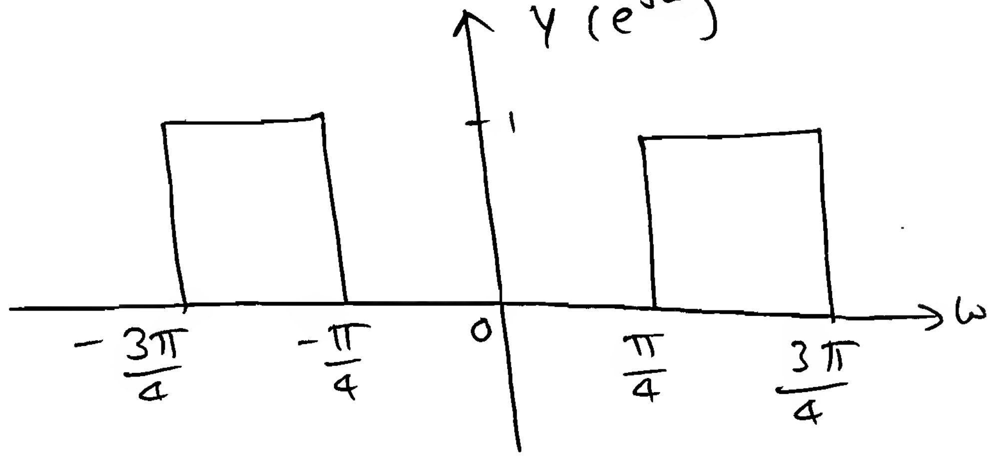
* Contn: $\frac{\sin at}{\pi t} \xleftrightarrow{F.T.}$



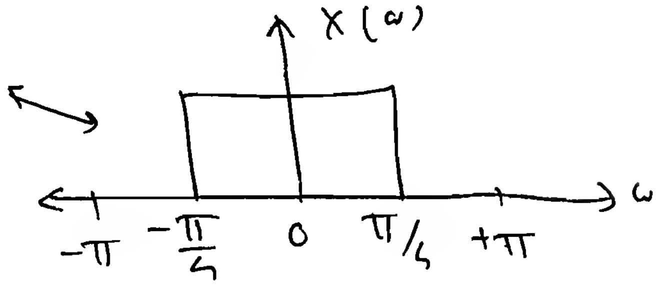
Disr: $\boxed{\frac{\sin \omega_c n}{\pi n} \xleftrightarrow{F.T.}}$



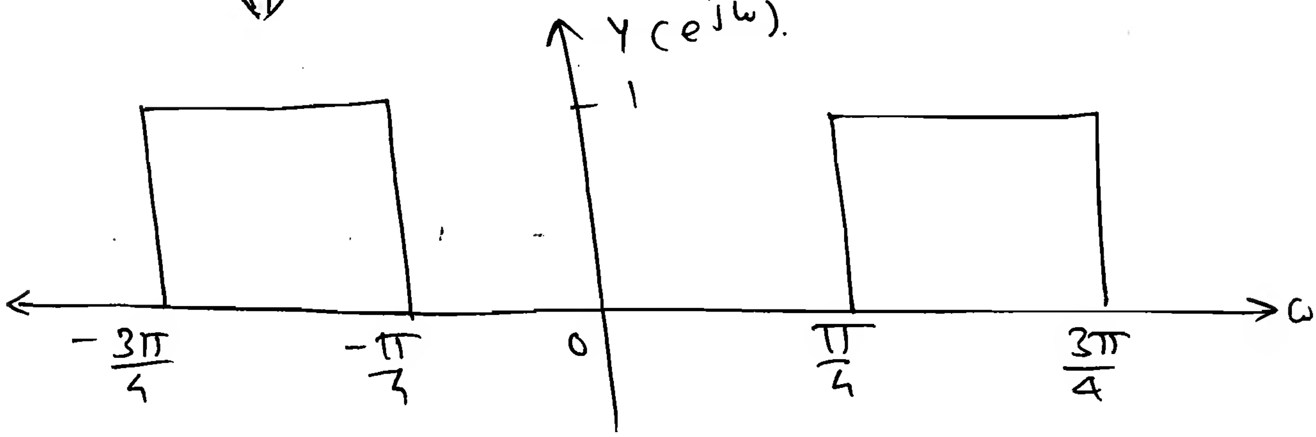
P 6.1.4 Find the signal corresponding to the spectrum shown in figure?



Soln: let, $x(n) = \dfrac{\sin \frac{\pi}{4} n}{\pi n} \longleftrightarrow$



$\Downarrow$



$$\therefore \quad Y(e^{j\omega}) = X\left[e^{+j(\omega - \frac{\pi}{2})}\right] + X\left[e^{j(\omega + \frac{\pi}{2})}\right].$$

$\downarrow$ I.F.T

$$y(n) = x(n) \cdot e^{j\frac{\pi}{2}n} + x(n) \cdot e^{-j\frac{\pi}{2}n}$$

$$\therefore \quad y(n) = x(n) \cdot e^{+j\frac{\pi}{2}n} + x(n) \cdot e^{-j\frac{\pi}{2}n}$$

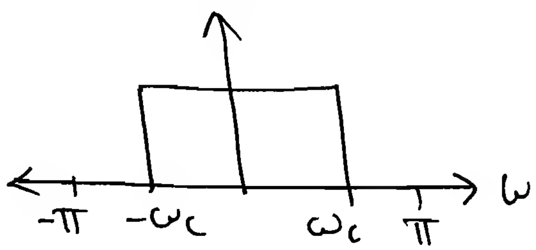
$$\boxed{y(n) = 2x(n) \cdot \cos\left(\frac{\pi}{2}n\right).}$$

$\boxed{P\ 6.1.5}$ (a) Let $h(n)$ is the impulse response of ideal L.P.F. with cutoff frequency ω_c, what type of filter has unit impulse response as $g(n) = (-1)^n . h(n)$.

Sol:

$$h(n) \longleftrightarrow H(\omega).$$

$$\therefore h(n) = \frac{\sin \omega_c n}{n\pi} \xleftrightarrow{F.T.}$$



$\Rightarrow \quad g(n) = (-1)^n . h(n).$

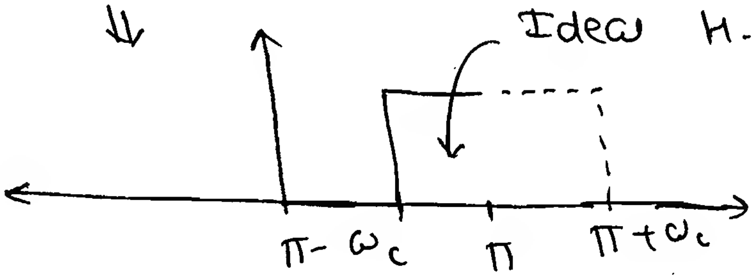
$$g(n) = e^{jn\pi} . h(n).$$

$\downarrow$ F.T.

$$G(e^{j\omega}) = H(e^{j(\omega-\pi)}).$$

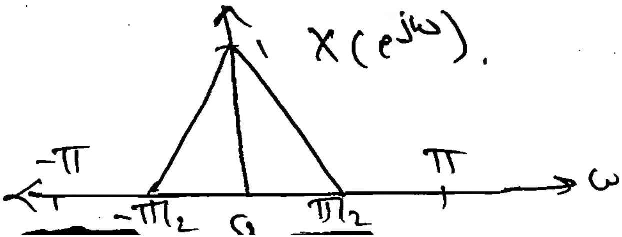
$\Downarrow$

$=$



$\Rightarrow$ Type of filter is Ideal HPF.

(b) A discrete system with input $x(n)$ & output $y(n)$ are related as $y(n) = x(n) + (-1)^n x(n)$. If the input spectrum $X(e^{j\omega})$ is shown below the o/p spectrum at $\omega = 0$ & $\omega = \pi$ are.

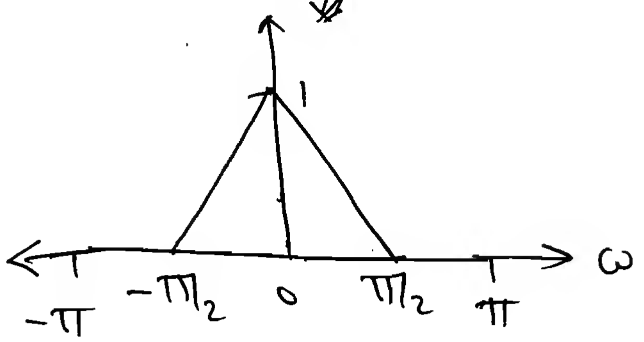


Soln:
$$y(n) = x(n) + (-1)^n \cdot x(n).$$

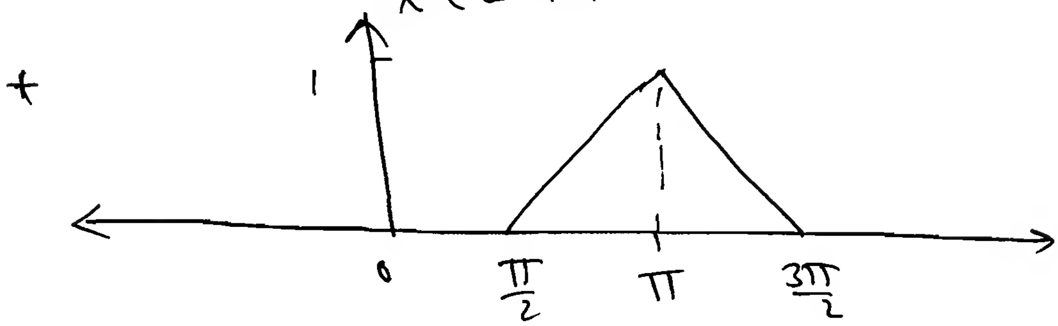
$$\Rightarrow y(n) = x(n) + e^{jn\pi} \cdot x(n).$$

$\downarrow$ F.T.

$$\therefore Y(e^{j\omega}) = X(e^{j\omega}) + X(e^{j(\omega-\pi)}).$$



X(ω−π).



so, The o/p spectrum i.e.

$Y(e^{j\omega})$ at $\omega = 0$ is 1 and

$\omega = \pi$ is also 1.

* Time-Scaling :-

$\Rightarrow$ $\boxed{x(n) \longleftrightarrow X(e^{j\omega})}$, then $x[n/k] \longleftrightarrow X(e^{j\omega k})$

then $\boxed{x(n/k) \longleftrightarrow X(e^{j\omega k}).}$

$\Rightarrow$ n is integer multiple of k.

P 6.1.6. Find the I.F.T. of $Y(e^{j\omega})$

$$Y(e^{j\omega}) = \frac{1}{1 - \frac{1}{2} \cdot e^{-j10\omega}}$$

Soln:

$$Y(e^{j\omega}) = \frac{1}{1 - \frac{1}{2} \cdot e^{-j10\omega}} \quad k = 10.$$

$\downarrow$ I.F.T.

$$\therefore y(n) = \left(\frac{1}{2}\right)^{\frac{n}{10}} \cdot u\left(\frac{n}{10}\right), \quad n = 0, 10, 20, \ldots$$

P 6.1.7. If the DTFT of $x(n) = \left(\frac{1}{5}\right)^n u(n+2)$ is $X(e^{j\omega})$, find the sequence that has a DTFT given $Y(e^{j\omega}) = X(e^{j2\omega})$.

Soln:

$$Y(e^{j\omega}) = X(e^{j2\omega}) \quad k = 2$$

$\downarrow$ I.F.T.

$$\therefore y(n) = x(n/k) = x(n/2)$$

$$\therefore \boxed{y(n) = \left(\frac{1}{5}\right)^{n/2} \cdot u\left(\frac{n}{2} + 2\right), \quad n = 0, 2, 4, 6, \ldots}$$

* Frequency Differentiation :-

$$\Rightarrow \boxed{-jn\,x(n) \longleftrightarrow \frac{d}{d\omega} X(e^{j\omega})}$$

$$\Rightarrow \boxed{n\,x(n) \longleftrightarrow j\frac{d}{d\omega}X(e^{j\omega})}$$

P 6.1.8. Find the F.T. of $y(n) = n\,a^n.u(n)$?

Soln: Let, $x(n) = a^n.u(n)$.

$\downarrow$ F.T.

$$\therefore X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$$

$$\therefore n\,a^n.u(n) \longleftrightarrow j\frac{d}{d\omega}\left(X(e^{j\omega})\right).$$

$$Y(e^{j\omega}) = j\frac{d}{d\omega}\left[\frac{1}{1-ae^{-j\omega}}\right].$$

$$= j\,\frac{-a(-j).e^{-j\omega}}{(1-ae^{-j\omega})^2}$$

$$Y(e^{j\omega}) = \frac{j^2 a.e^{-j\omega}}{(1-ae^{-j\omega})^2}$$

$$\boxed{Y(e^{j\omega}) = \frac{-a.e^{-j\omega}}{(1-ae^{-j\omega})^2}}$$

P. 6.1.9. Find the value of $\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n$?

Soln: $$\sum_{n=0}^{\infty} n\left(\frac{1}{a}\right)^n = \left.\frac{-a.e^{-j\omega}}{(1-ae^{-j\omega})^2}\right|_{\omega=0} = \frac{-a.e^{-j0}}{(1-ae^{-j(0)})^2}$$

$$\Rightarrow \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = \frac{-\frac{1}{2}}{(1-\frac{1}{2})^2}.$$

$$\sum_{n=0}^{\infty} n \cdot \left(\tfrac{1}{2}\right)^n = \frac{-\frac{1}{2}}{\left(\frac{1}{2}\right)^2} = -2.$$

$\boxed{\text{P 6.1.10}}$ Find the F.T. of $x(n) = n e^{\frac{jn\pi}{8}} \cdot \alpha^{n-3} \cdot u(n-3)$.

Soln:

$$x(n) = n \cdot e^{\frac{jn\pi}{8}} \cdot \alpha^{(n-3)} u(n-3).$$

$\frac{\pi}{8} \rightarrow \omega_0$, $3 \rightarrow n_0$

$$= j \frac{d}{d\omega} \left[\frac{e^{-j\left(\omega - \frac{\pi}{8}\right) \cdot 3}}{1 - \alpha e^{-j\left(\omega - \frac{\pi}{8}\right)}} \right]$$

* Convolution in time :-

$\Rightarrow$ $x(n) \longleftrightarrow X(e^{j\omega})$ &

$h(n) \longleftrightarrow H(e^{j\omega})$. then

$$x(n) * h(n) \longleftrightarrow X(e^{j\omega}) \cdot H(e^{j\omega}).$$

Note:-

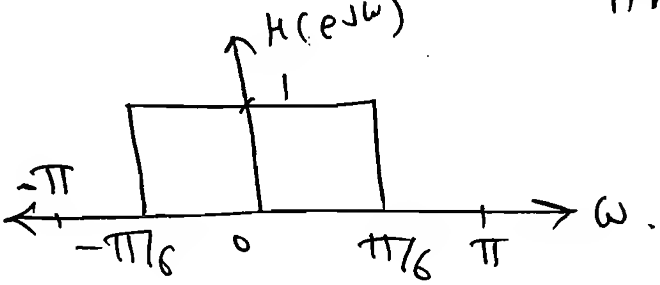
$\Rightarrow$ F.T. of impulse response is known as frequency response.

$\boxed{\text{P 6.1.11}}$ Consider $x(n) = \sin\left(\frac{n\pi}{8}\right) - 2\cos\left(\frac{\pi n}{4}\right)$.

if the impulse response is $h(n) = \dfrac{\sin\left(\frac{\pi n}{6}\right)}{\pi n}$.

Soln:

$$h(n) = \frac{\sin \frac{n\pi}{6}}{\pi n} \overset{F.T}{\longleftrightarrow}$$



So, only $\sin\left(\frac{\pi n}{8}\right)$ will be passed through $h(n)$.

hence O/P is $\boxed{y(n) = \sin\left(\frac{\pi n}{8}\right)}$.

$\boxed{\text{P 6.1.12}}$ An L.T.I. System is having impulse response

$$h(n) = \begin{cases} 4\sqrt{2} & ; n = 2, -2 \\ -2\sqrt{2} & ; n = 1, -1 \\ 0 & ; \text{elsewhere.} \end{cases}$$

Find the output when input applied is $x(n) = e^{jn\pi/4}$?

Soln:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) \cdot e^{-j\omega n}.$$

$$H(e^{j\omega}) = 4\sqrt{2}\left[e^{-j2\omega} + e^{j2\omega}\right] + (-2\sqrt{2})\left[e^{j\omega} + e^{-j\omega}\right].$$

$$H(e^{j\omega}) = 8\sqrt{2}\cos 2\omega - 4\sqrt{2}\cos\omega.$$

Now, $x(n) = e^{jn\pi/4}$.

$\therefore$ if i/p is $x(n) = e^{j\omega n} \Rightarrow y(n) = e^{j\omega n} \cdot H(\omega)$.

$$\Rightarrow x(n) = e^{jn\pi/4} \Rightarrow y(n) = e^{jn\pi/4} \cdot H(\omega)\Big|_{\omega = \pi/4}.$$

$$= \left[0 - 4\sqrt{2} \times 1/\sqrt{2}\right] e^{\frac{jn\pi}{4}}$$

$$-4 \cdot e^{jn\pi/4}$$

$$\Rightarrow \boxed{y(n) = 4.\, e^{jn\pi/4}}$$

$\Rightarrow$ If $\quad x(n) = (-1)^n = e^{jn\pi}$

$$\Rightarrow y(n) = e^{jn\pi}\left[8\sqrt{2}\cos 2\pi - 4\sqrt{2}\cos\pi\right].$$

$$= e^{jn\pi}\left[8\sqrt{2}(1) - 4\sqrt{2}(-1)\right].$$

$$\boxed{y(n) = 12\sqrt{2}.\, e^{jn\pi}}$$

P 6.1.13 Design a 3 point FIR filter with impulse response $h(n) = \{\alpha, \underset{\uparrow}{\beta}, \alpha\}$ & the magnitude response blocks the frequency $f = 1/3$ & phase the freq. $f = 1/8$ with unity gain. What is the D.C. gain of the filter?

Soln:

$$h(n) = \{\underset{n=-1}{\alpha}, \underset{\substack{\uparrow \\ n=0}}{\beta}, \underset{n=1}{\alpha}\}$$

$\downarrow$ F.T.

$$\Rightarrow H(e^{j\omega}) = \sum_{n=-1}^{+1} h(n).\, e^{-j\omega n}.$$

$$= \alpha e^{-j\omega(-1)} + \beta + \alpha e^{+j\omega(1)}.$$

$$\therefore \boxed{H(e^{j\omega}) = 2\alpha.\cos\omega + \beta.}$$

Now. given that $H(e^{j\omega})\Big|_{f=1/3} = 0.$

& $H(e^{j\omega})\Big|_{f=1/8} = 1.$

$$\Rightarrow H(e^{j\omega})\Big|_{\omega=2\pi\left(\frac{1}{3}\right)} = 0.$$

$$\Rightarrow 2\alpha.\cos\left(\frac{2\pi}{3}\right) + \beta = 0.$$

$$\Rightarrow -2\alpha.\cos\frac{\pi}{3} + \beta = 0.$$

$$\Rightarrow -2\alpha.\frac{1}{2} + \beta = 0 \Rightarrow \boxed{\alpha = \beta}.$$

$$\Rightarrow H(e^{j\omega})\Big|_{\omega=2\pi\left(\frac{1}{8}\right)} = 1.$$

$$\Rightarrow 2\alpha.\cos\left(\frac{\pi}{4}\right) + \beta = 1.$$

$$2\alpha.\frac{1}{\sqrt{2}} + \alpha = 1.$$

$$\boxed{\alpha = \frac{1}{1+\sqrt{2}} = \beta.}$$

So, $H(e^{j\omega}) = \frac{2}{1+\sqrt{2}}.\cos\omega + \frac{1}{1+\sqrt{2}}.$

$$\boxed{\text{D.C. gain} \Rightarrow H(e^{j\omega})\Big|_{\omega=0} = \frac{2}{1+\sqrt{2}} + \frac{1}{1+\sqrt{2}} = \frac{3}{1+\sqrt{2}}}$$

$\boxed{\text{P 6.1.14}}$ <u>Digital filters</u>:-

(1) $y(n) = x(n) - x(n-1)$. [HPF].

$\Rightarrow \downarrow$ F.T.

$$Y(e^{j\omega}) = X(e^{j\omega}) - e^{-j\omega}.X(e^{j\omega}).$$

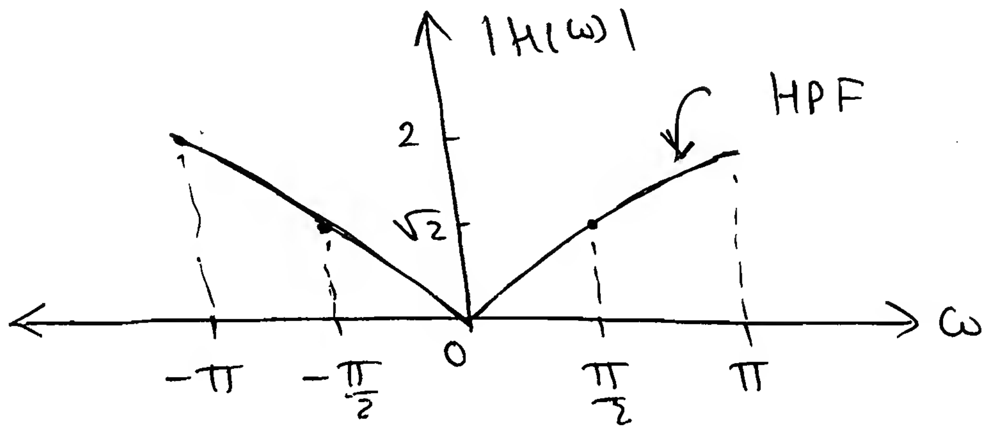
$$\Rightarrow \quad Y(\omega) = (1-e^{-j\omega})\, X(\omega).$$

$$\Rightarrow \text{T.F.} \quad H(\omega) = \frac{Y(\omega)}{X(\omega)} = 1-e^{-j\omega}.$$

$$\Rightarrow \omega = 0 \Rightarrow H(0) = 1-1 = 0.$$

$$\rightarrow \omega = \frac{\pi}{2} \Rightarrow H\left(\frac{\pi}{2}\right) = 1-e^{-j\frac{\pi}{2}} = 1+j = \sqrt{2}\angle 45^\circ$$

$$\rightarrow \omega = \pi \Rightarrow H(\pi) = 1-e^{-j\pi} = 1-(-1) = 2.$$



(2) $y(n) = x(n) + x(n-1).$ [LPF].

$$\Rightarrow \quad \downarrow \text{F.T.}$$

$$Y(\omega) = X(\omega) + e^{-j\omega} \cdot X(\omega).$$

$$\Rightarrow \text{T.F.} \quad \frac{Y(\omega)}{X(\omega)} = H(\omega) = 1+e^{-j\omega}.$$

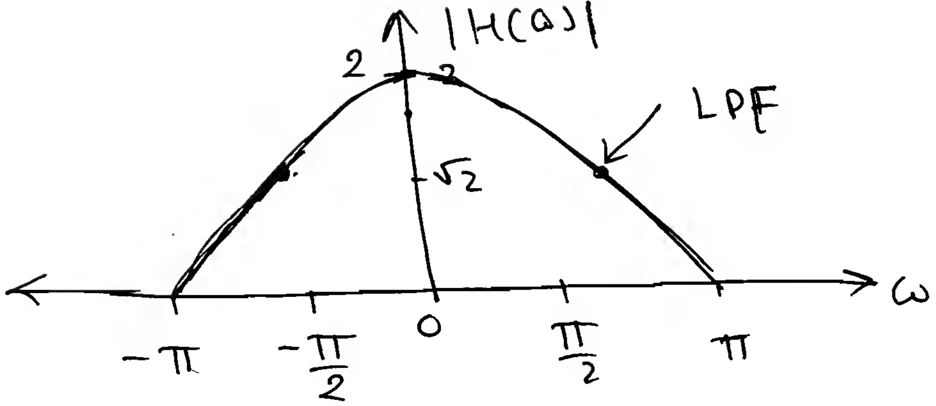
$$\rightarrow \omega = 0 \Rightarrow H(0) = 1+e^{-j0} = 2.$$

$$\rightarrow \omega = \frac{\pi}{2} \Rightarrow H\left(\frac{\pi}{2}\right) = 1+e^{-j\frac{\pi}{2}} = 1-j = \sqrt{2}\angle -45^\circ.$$

$$\rightarrow \omega = \pi \Rightarrow H(\pi) = 1+e^{-j\pi} = 1+(-1) = 0.$$

$$\Rightarrow \omega = -\pi \Rightarrow H(-\pi) = 1+e^{j\pi} = 1-1 = 0.$$

$\Rightarrow$



[3] $h(n) = \delta(n) - \delta(n-2)$. [<u>BPF</u>].

Soln: $\downarrow$ F.T.

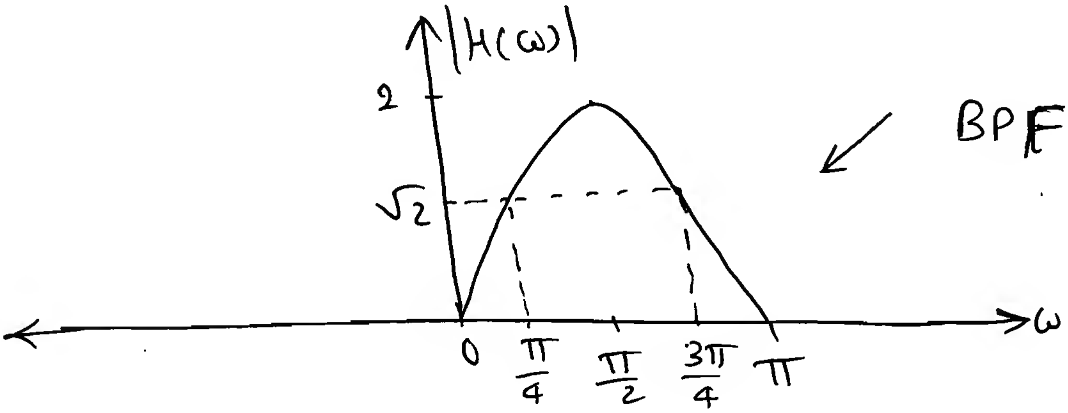
$\therefore H(\omega) = 1 - e^{-j2\omega} \cdot 1.$

$\Rightarrow H(\omega) = 1 - e^{-j2\omega}.$

$\rightarrow \omega = 0 \Rightarrow H(0) = 1 - e^{-j(0)} = 1-1 = 0.$

$\rightarrow \omega = \frac{\pi}{2} \Rightarrow H(\frac{\pi}{2}) = 1 - e^{-j2(\frac{\pi}{2})} = 1 - e^{-j\pi} = 1-(-1) = 2.$

$\Rightarrow \omega = \pi \Rightarrow H(\pi) = 1 - e^{-j2\pi} = 1 - 1 = 0.$



[4] $h[n] = \delta[n] + \delta[n+2]$.

Soln: $\downarrow$ F.T.

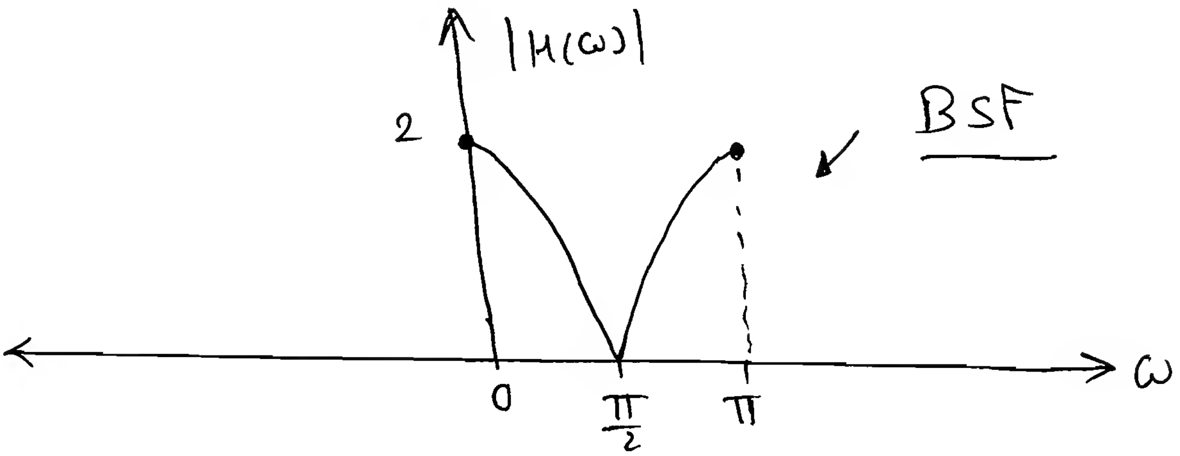
$\therefore H(\omega) = 1 + e^{-j2\omega} \cdot 1.$

$\Rightarrow \boxed{H(\omega) = 1 + e^{-j2\omega}}.$

$\rightarrow \omega = 0 \Rightarrow H(0) = 1 + e^{-j(0)} = 2.$

$\rightarrow \omega = \frac{\pi}{2} \Rightarrow H(\frac{\pi}{2}) = 1 + e^{-j \cancel{2}\pi \times \frac{\pi}{\cancel{2}}} = 1 + (-1) = 0.$

$\rightarrow \omega = \pi \Rightarrow H(\pi) = 1 + e^{-j2\pi} = 1 + 1 = 2.$



$\boxed{P6.1.15.}$ Consider the system described by the equation $y(n) = a y(n-1) + b x(n) + x(n-1)$, where 'a' & 'b' are real, find the relation betn a & b such that $|H(e^{j\omega})| = 1$

Soln:

$$y(n) = a y(n-1) + b x(n) + x(n-1).$$

$\downarrow$ F.T.

$$\therefore Y(\omega) = a \cdot e^{-j\omega} Y(\omega) + b X(\omega) + e^{-j\omega} X(\omega).$$

$$\therefore Y(\omega)[1 - a e^{-j\omega}] = [b + e^{-j\omega}] X(\omega).$$

$$\therefore H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{b + e^{-j\omega}}{1 - a \cdot e^{-j\omega}}.$$

Now $|H(e^{j\omega})| = 1 \Rightarrow |H(\omega)|^2 = 1.$

$$\Rightarrow H(\omega) \cdot H^*(\omega) = 1.$$

$$\Rightarrow \left[\frac{b+e^{-j\omega}}{1-a e^{-j\omega}}\right]\cdot\left[\frac{b+e^{j\omega}}{1-a e^{j\omega}}\right] = 1.$$

$$\therefore \frac{b^2+b\left[e^{j\omega}+e^{-j\omega}\right]+1}{1-a\left[e^{-j\omega}+e^{j\omega}\right]+a^2} = 1.$$

$$\therefore b^2+2b\cos\omega+\cancel{1} = \cancel{1}-2a\cos\omega+a^2$$

$$\therefore a^2-b^2 = 2\cos\omega\,[b-a].$$

$$\therefore -(a+b) = 2\cos\omega.$$

$$\Rightarrow \boxed{a=-b}$$

(or) $$\left|H(e^{j\omega})\right|_{\omega=0} = 1. \qquad (\because =1\ \forall\omega).$$

$$\therefore \frac{b+1}{1-a} = 1.$$

$$b+\cancel{1} = \cancel{1}-a \Rightarrow \boxed{b=-a}$$

<u>DRDO</u>

$\boxed{P6.1.16}$ An input $x(n)$ with length 3 is applied to a LTI system having an impulse response $h(n)$ of length 5, and $Y(\omega)$ is the DTFT of the o/p $y(n)$ of the system. If $|h(n)|\le L$ & $|x(n)|\le B\ \forall n$, the maximum value of $Y(0)$ can be ___.

(a) 15LB (B) 12LB

(C) 8LB (D) 7LB.

$\underline{\underline{Sol^n}}$: $y(n) = x(n) * h(n)$.

$\Rightarrow Y(\omega) = X(\omega) \cdot H(\omega)$.

$\Rightarrow Y(0) = X(0) \cdot H(0)$.

$\Rightarrow Y(0) = \left[\sum_{n=0}^{2} x(n)\right] \times \left[\sum_{n=0}^{4} h(n)\right]$

$x(n) \rightarrow L = 3$

$\rightarrow X(\omega) = \sum_{n=0}^{2} x(n) \cdot e^{-j\omega n}$.

$X(0) = \sum_{n=0}^{2} x(n)$.

$\rightarrow H(\omega) = \sum_{n=0}^{4} h(n) \cdot e^{-j\omega n}$

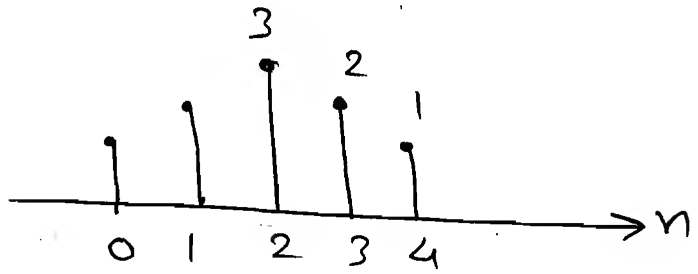
$\rightarrow H(0) = \sum_{n=0}^{4} h(n)$.

$\Rightarrow Y(0) = \sum_{n=0}^{2} B \times \sum_{n=0}^{4} L$.

$Y(0) = 3B \times 5L$

$\Rightarrow \boxed{Y(0) = 15LB}$

P 6.1.17 Consider a filter with I.R. shown in figure. Find the group delay of the filter?



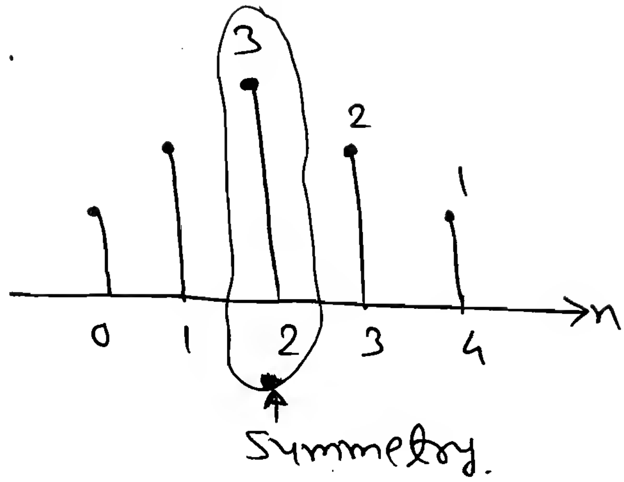
$\underline{\underline{Sol^n}}$: $\underline{\text{F.I.R.}}$ $\underline{\text{filters}}$:

Linear phase response $\theta(\omega) = -\alpha\omega$.

$\therefore$ α is the value of n for which Spectrum is symmetrical about n.

In this case, $n = 2$.

$\Rightarrow \quad \theta(\omega) = -2\omega$



$\Rightarrow \quad t_g(\omega) = -\dfrac{d\theta(\omega)}{d\omega}$.

$\Rightarrow \quad \boxed{t_g(\omega) = +2}$

$\Rightarrow \quad h[n] = \pm h[N-1-n]$. $\Rightarrow$ For symmetry.

here, N = length of I.R. = 5.

$\therefore \quad h[n] = h[4-n]$.

$\Rightarrow \quad h(0) = h(4) = 1$.

$h(1) = h(3) = 2$.

*** Note:**

> Group delay $t_g(\omega)$ = Value of n about which spectrum is symmetrical.

P 6.1.18 An L.T.I. filter is described by the difference equation $y(n) = x(n) + 2x(n-1) + x(n-2)$.

(a) Obtain the magnitude & phase response ?

(b) Find the o/p when the input is

$$x(n) = 10 + 4\cos\left[\frac{\pi n}{2} + \frac{\pi}{4}\right] ?$$

Soln: $y(n) = x(n) + 2x(n-1) + x(n-2)$.

$\xrightarrow{F.T.} \quad Y(\omega) = X(\omega) + 2e^{-j\omega} \cdot X(\omega) + 2e^{-j2\omega} \cdot X(\omega)$.

$$\Rightarrow \quad H(\omega) = \frac{Y(\omega)}{X(\omega)} = 1 + 2e^{-j\omega} + e^{-j2\omega}$$

$$H(\omega) = (1 + 2\cos\omega + \cos 2\omega) - j(\sin\omega + \sin 2\omega)$$

Mag.

$$\Rightarrow \quad |H(\omega)| = \sqrt{(1+2\cos\omega+\cos 2\omega)^2 + (\sin\omega + \sin 2\omega)^2}$$

Phase

$$\angle H(\omega) = -\tan^{-1}\left(\frac{\sin\omega + \sin 2\omega}{1+2\cos\omega+\cos 2\omega}\right).$$

(b) $x(n) = 10 + 4\cos\left[\frac{\pi}{2}n + \frac{\pi}{4}\right]$?

$$\Rightarrow \quad H(\omega) = (1+e^{-j\omega})^2.$$

$$= e^{-j\frac{\omega}{2}(2)}\left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right)^2$$

$$= 4e^{-j\omega}\left(\frac{e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}}{2}\right)^2$$

$$H(\omega) = 4e^{-j\omega} \cdot \cos^2\frac{\omega}{2}.$$

$\Rightarrow$ Mg. $|H(\omega)| = 4.\cos^2\frac{\omega}{2}.$

Phase $\angle H(\omega) = \theta(\omega) = -\omega.$

Now $x(n) = \underbrace{10}_{\omega=0} + \underbrace{4\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}_{\omega=\pi/2.}$

$H(\omega)\big|_{\omega=0} = 4.$, $H(\omega)\big|_{\omega=\frac{\pi}{2}} = 4.e^{-j\frac{\pi}{2}} \cdot \cos^2\frac{\pi}{4}$

$= 2.e^{-j\pi/2}.$

$\therefore \quad y(n) = K \cdot A \cdot \cos(\omega_0 n + \phi)$.

$\Rightarrow y(n) = 40 + 4(2) \cdot \cos\left[\frac{\pi n}{2} + \frac{\pi}{4} - \frac{\pi}{2}\right]$.

$$\Rightarrow \boxed{y(n) = 40 + 8 \cdot \cos\left[\frac{\pi n}{2} - \frac{\pi}{4}\right]}$$

* Parseval's relation :-

$\Rightarrow \quad x(n) \longleftrightarrow X(e^{j\omega})$.

the $$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} |X(e^{j\omega})|^2 \, d\omega.$$

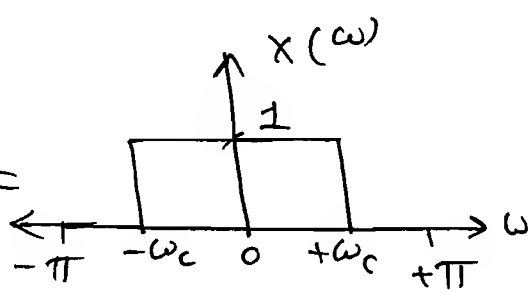
$\Rightarrow$ Parseval's relation is known as Conservation of energy theorem, because DTFT operator preserves energy, when going from time domain to freq. domain.

$\boxed{P\ 6.1.20}$ Find the energy in the signal

$x(n) = \dfrac{\sin \omega_c n}{\pi n}$.

Soln:

$x(n) = \dfrac{\sin \omega_c n}{\pi n} \xleftrightarrow{F.T.} X(\omega) =$



$$\therefore E_{x(n)} = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \cdot d\omega.$$

$$\Rightarrow \quad E_{x(n)} = \frac{1}{2\pi}\int_{-\omega_c}^{+\omega_c} (1)^2 \cdot d\omega .$$

$$= \frac{2\omega_c}{2\pi} = \frac{\omega_c}{\pi} .$$

$$\Rightarrow \quad \boxed{E_{x(n)} = \frac{\omega_c}{\pi}} .$$

P 6.1.21 Find the value of

$$\sum_{n=-\infty}^{+\infty} \frac{\sin\left(\frac{n\pi}{4}\right) \cdot \sin\left(\frac{n\pi}{3}\right)}{2\pi n \cdot 5\pi n} .$$

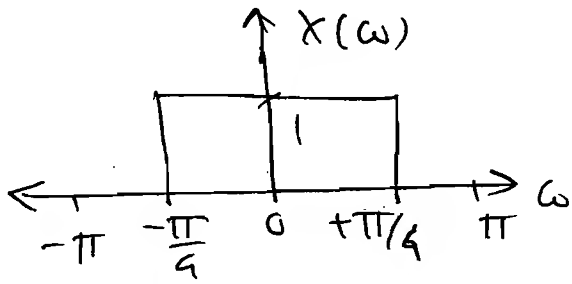
Soln:

$$\boxed{\sum_{n=-\infty}^{\infty} x[n] \cdot y[n] = \frac{1}{2\pi}\int_{\langle 2\pi \rangle} X(e^{j\omega}) \cdot Y(e^{j\omega}) \cdot d\omega}$$

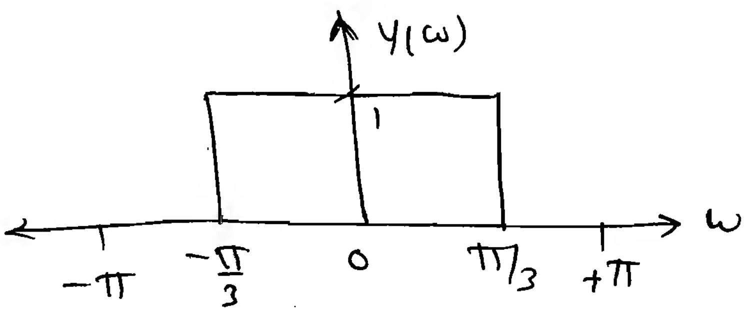
$\downarrow$

Planchar's Relation.

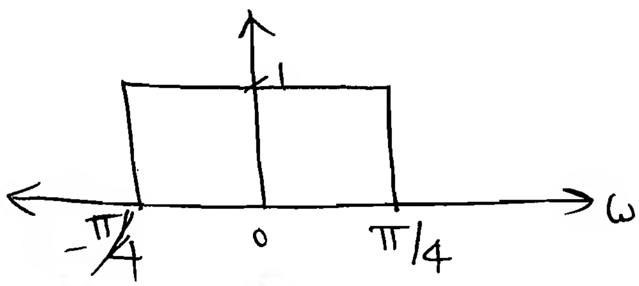
let, $x_1(n) = \dfrac{\sin \frac{n\pi}{4}}{\pi n} \longleftrightarrow$



$Y(n) = \dfrac{\sin\left(\frac{n\pi}{3}\right)}{\pi n} \longleftrightarrow$



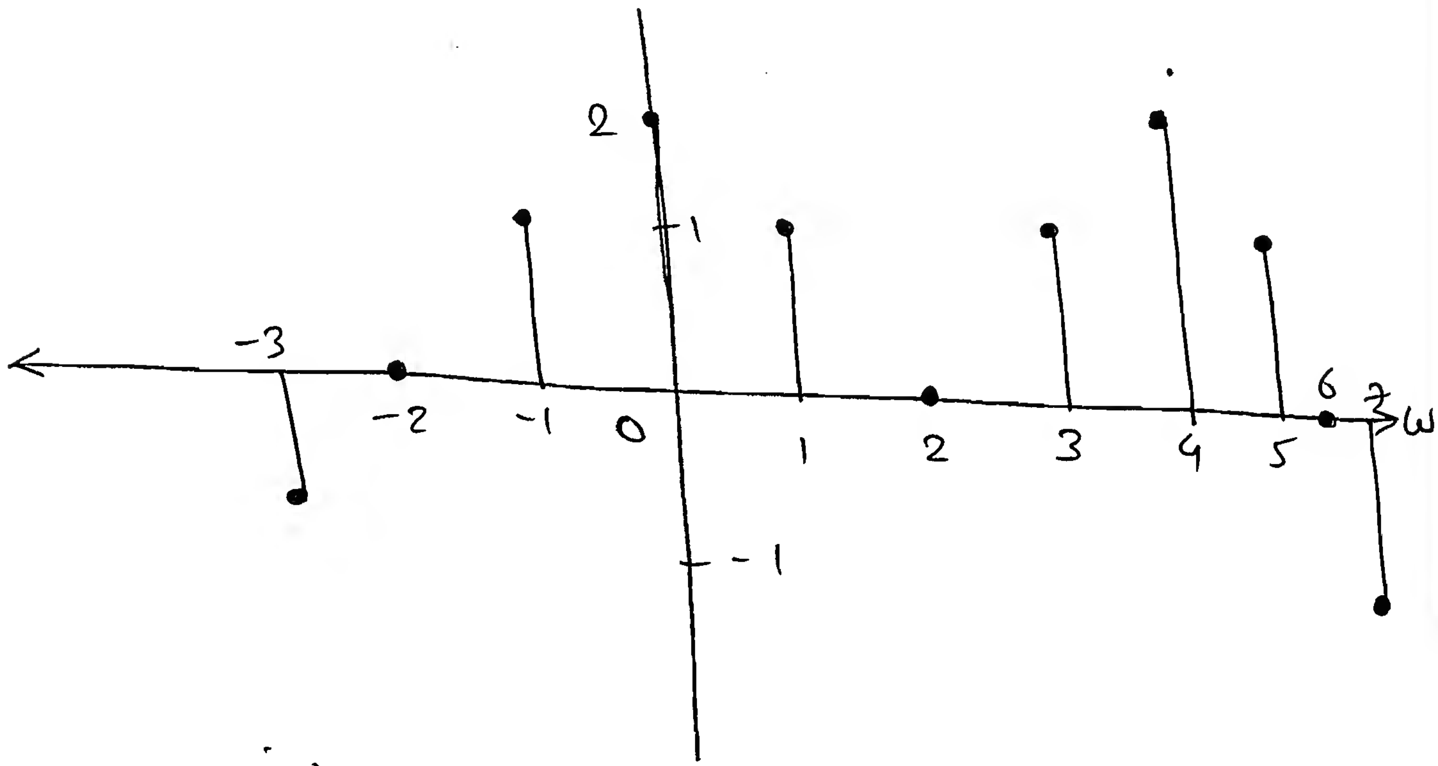
$X(\omega) \cdot Y(\omega) =$



$$\therefore \sum_{n=-\infty}^{+\infty} \frac{\sin\left(\frac{n\pi}{4}\right)}{2\pi n} \cdot \frac{\sin\left(\frac{n\pi}{3}\right)}{5\pi n} = \frac{1}{2\pi \times 10} \int_{-\pi/4}^{\pi/4} (1) \cdot d\omega.$$

$$= \frac{1}{2\cancel{\pi} \times 10} \times \frac{\cancel{\pi}}{2} = \boxed{\frac{1}{40}}$$

$\boxed{\text{P 6.1.22}}$ For the signal shown in fig 6.1.22, find the following quantities without calculating DTFT?



(a) $X(e^{j0})$

Sol^n^:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n) \cdot e^{-j\omega n}$$

$$X(e^{j0}) = \sum_{n=-\infty}^{\infty} x(n) \cdot (1).$$

$$X(e^{j0}) = \sum_{n=-3}^{7} x(n). = x(-3) + x(-2) + x(-1) + x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7).$$

$\therefore X(e^{j0}) = -\cancel{1} + 0 + \cancel{1} + 2 + 1 + 0 + 1 + 2 + \cancel{1} + 0 - \cancel{1}$

$$\boxed{X(e^{j0}) = 6}$$

(b) $X(e^{j\pi})$.

Soln:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n) \cdot e^{-j\omega n}$$

$$\therefore X(e^{j\pi}) = \sum_{n=-\infty}^{+\infty} x(n) \cdot e^{-j\pi n} = \sum_{n=-\infty}^{+\infty} x(n) \cdot (-1)^n.$$

$$\therefore X(e^{j\pi}) = -x(-3) + x(-2) - x(-1) + x(0) - x(1) + x(2) - x(3) + x(4) - x(5) + x(6) - x(7).$$

$$= +\cancel{1} - \cancel{1} + 2 - 1 - 1 + 2 - \cancel{1} + \cancel{1}$$

$$\boxed{X(e^{j\pi}) = 2}$$

(c) $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$.

$$\Rightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n) \cdot e^{-j\omega n}$$

$$\therefore \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot d\omega = \int_{-\pi}^{\pi} \sum_{n=-\infty}^{+\infty} x(n) \cdot [\cos\omega n - j\sin\omega n]\, d\omega.$$

$$\Rightarrow x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot e^{j\omega n} d\omega.$$

Put $n = 0$.

$$\Rightarrow \boxed{\int_{-\pi}^{+\pi} X(e^{j\omega}) \cdot (1) \cdot d\omega = 2\pi x(0)}$$

so,
$$\int_{-\pi}^{+\pi} X(e^{j\omega}) \cdot d\omega = 2\pi(2) = 4\pi.$$

(d) $\int_{-\pi}^{+\pi} X(e^{j\omega}) \cdot e^{j2\omega} \cdot d\omega.$

Soln:
$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot e^{j2\omega n} \cdot d\omega.$$

here, $n=2$.

$$\therefore \int_{-\pi}^{+\pi} X(e^{j\omega}) e^{j2\omega} \cdot d\omega = 2\pi\ x(2) = 0.$$

(e) $\int_{-\pi}^{+\pi} |X(e^{j\omega})|^2 d\omega.$

Soln:
$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega.$$

$$\Rightarrow \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 2\pi \left[\sum_{n=-\infty}^{\infty} |x(n)|^2 \right].$$

$$= 2\pi [1+1+4+1+1+4+1+1].$$

$$= 28\pi.$$

(f) $\int_{-\pi}^{\pi} \left| \frac{d}{d\omega} X(e^{j\omega}) \right|^2 d\omega.$

Soln:

$x(n) \longleftrightarrow X(e^{j\omega})$.

$n x(n) \longleftrightarrow +j \frac{d}{d\omega} X(e^{j\omega})$.

$$\therefore \int_{-\pi}^{+\pi} \left| \frac{d}{d\omega} X(e^{j\omega}) \right|^2 d\omega = 2\pi \sum_{n=-\infty}^{+\infty} |-jn\, x(n)|^2.$$

$$= 2\pi \sum_{n=-\infty}^{+\infty} |n x(n)|^2.$$

$$= 2\pi [9 + 0 + 1 + 0 + 1 + 0 + 1 + 64 + 25 + 49].$$

$$= 300\pi$$

(g) $\angle X(e^{j\omega})$.

Soln: $\angle X(e^{j\omega}) = \theta(\omega) = -2\omega = -2\omega$.

($\because$ Symm. @ $n=2$).

$\boxed{\text{P 6.1.23}}$ Given $h(n) = [1, 2, 2]$, $f(n)$ is obtained by convolving $h(n)$ with itself and $g(n)$ by correlating $h(n)$ with itself. Which one of the following statements is TRUE?

(a) $f(n)$ is causal and its maximum value is 9.

(b) $f(n)$ is non-causal.

(c) $g(n)$ is causal and its maximum

value is 9.

(d) $g(n)$ is non causal and maximum value is 9.

Soln: $f(n) = h(n) * h(n)$. ← ~~Correlation~~ convolution

$g(n) = h(n) * h(-n)$. ← correlation

$\Rightarrow h(n) = [1, 2, 2] \rightarrow 0 \le n \le 2.$

$n \to 0, 1, 2$

$\Rightarrow h(-n) = [1, 2, 2] \rightarrow -2 \le n \le 0.$

$n \to +2, -1, 0$

<u>Limits</u>

$\Rightarrow f(n) = \underset{0 \le n \le 2}{h(n)} * \underset{0 \le n \le 2}{h(n)} \Rightarrow 0 \le n \le 4. \rightarrow$ C

$\Rightarrow g(n) = \underset{0 \le n \le 2}{h(n)} * \underset{-2 \le n \le 0}{h(-n)} \longrightarrow -2 \le n \le 2. \rightarrow$ NC

Now, $f(n) = h(n) * h(n)$

$\Rightarrow f(n) = \{1, 4, 10, 8, 4\}.$

	1	2	2
1	1	2	2
2	2	4	4
2	2	4	4

$\Rightarrow g(n) = h(n) * h(-n).$

$= \{2, 6, 9, 8, 2\}.$

	1	2	2
2	2	4	4
2	2	4	4
1	1	2	2

So, Ans - (d) $g(n)$ is N.C. and max. value is 9.

P6.1.14 A continuous time signal $x(t)$ is to be filtered to remove freq. component in the range $5\ \text{kHz} \le f \le 10\ \text{kHz}$. The maximum freq. present in $x(t)$ is 20 kHz. Find the minimum sampling frequency & find freq. response of ideal digital filter that will remove the desired freq. from $x(t)$?

⇒ Soln:

$5\ \text{kHz} \le f \le 10\ \text{kHz}$ ← Analog range.

⇒ $-\pi \le \omega \le +\pi$ ← Digital filter.

↳ Dig. freq.

⇒ here, $f_{max} = 20$ kHz.

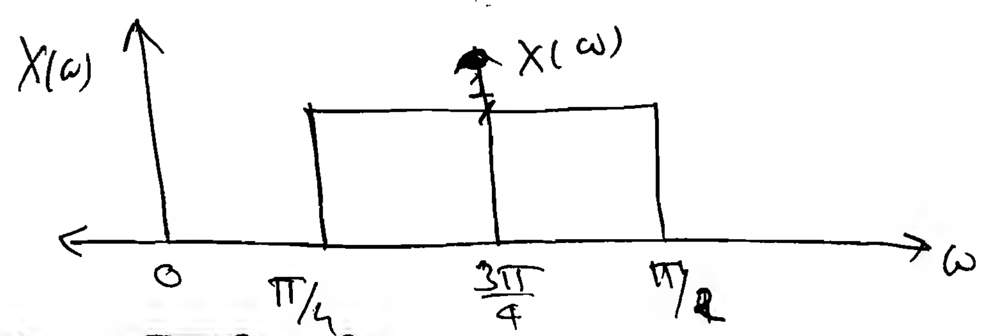
⇒ S.f. ⇒ $f_s = 2 f_m = 40$ kHz.

**

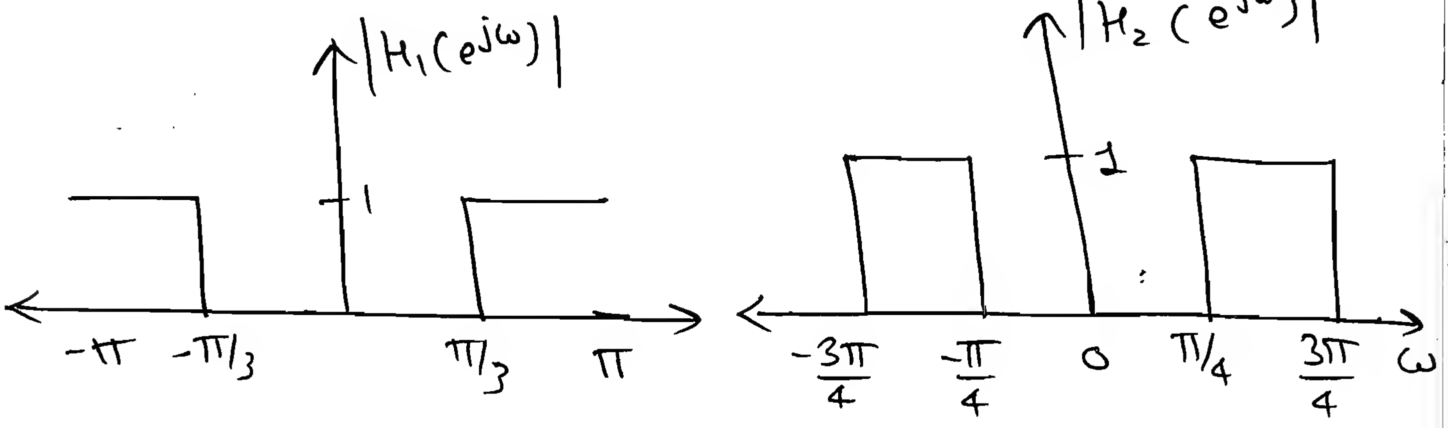
* Digital freq. $\omega = \dfrac{2\pi f}{f_s}$ → Analog freq., f_s → sampling freq.

$\therefore \dfrac{2\pi(5k)}{40k} \le \omega \le \dfrac{2\pi(10k)}{40k}$.

$\therefore \boxed{\dfrac{\pi}{4} \le \omega \le \dfrac{\pi}{2}}$



P 6.1.25 2 ideal filters freq. response are shown in fig. For an arbitrary input $x(n)$, find the range of freq. that can be present in the o/p $y(n)$, if they are connected in (a) Cascade (b) Parallel.



Soln:

(a) Cascade:

$$|H(e^{j\omega})| = |H_1(e^{j\omega})| \cdot |H_2(e^{j\omega})|.$$

so, $\boxed{\pi/3 \leq \omega \leq \frac{3\pi}{4}}$.

(b) Parallel.

$$|H(e^{j\omega})| = |H_1(e^{j\omega})| + |H_2(e^{j\omega})|.$$

so, $\boxed{\omega \geq \frac{\pi}{4}}$

*

$$e^{-at} * e^{-at} = te^{-at} \longleftrightarrow \frac{1}{(a+j\omega)^2}.$$

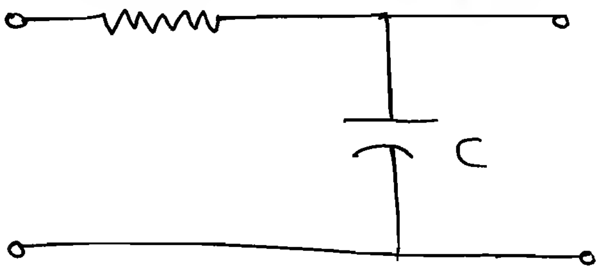
$$a^n u[n] * a^n u[n] \longleftrightarrow \frac{1}{(1-ae^{-j\omega})^2}.$$
$$= (n+1)\, a^n u[n]$$

Ch-7 - Z- Transform

⇒ Generation (or) Generalization of DTFT is Z - transform.

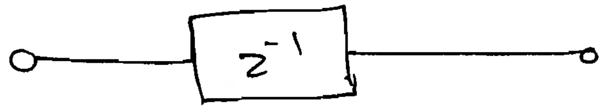
⇒ Discrete version of L.T.

⇒ R= 1KΩ ± 2% tolerance



$$H(s) = \frac{1}{1+SCR}.$$

⇓



⇒ $S = \sigma + j\omega$

↓

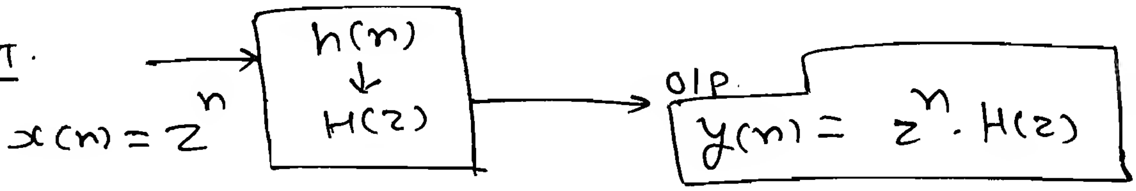
Complex variable

↓

$\boxed{z = r e^{j\omega}}$

⇒ In L.T. if i/p $x(t) = e^{st}$ ⇒ $\boxed{y(t) = e^{st} . H(s)}$

⇒ In Z.T.



⇒ Z.T. of general D.T. signal x(n) is

$$\boxed{X(z) = \sum_{n=-\infty}^{+\infty} x(n) . z^{-n}}$$

Now $z = r \cdot e^{j\omega}$.

$$\therefore X(r \cdot e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left[x[n] \cdot r^{-n} \right] e^{-j\omega n}$$

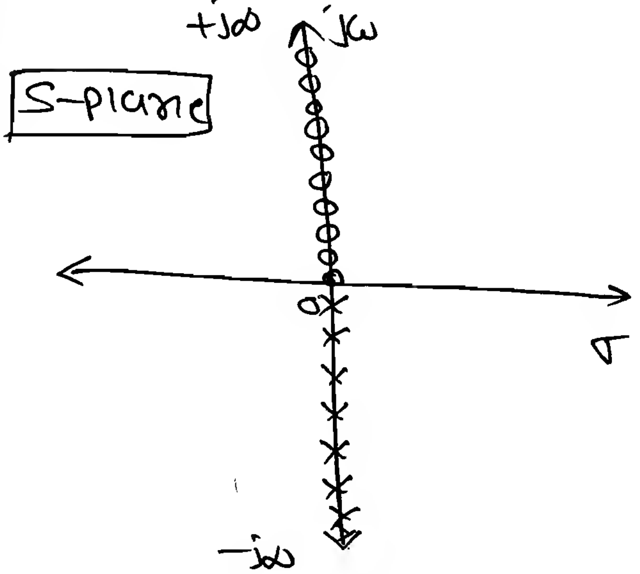
$$\Rightarrow \boxed{X(z) = F.T. \{ x(n) \cdot r^{-n} \}}$$

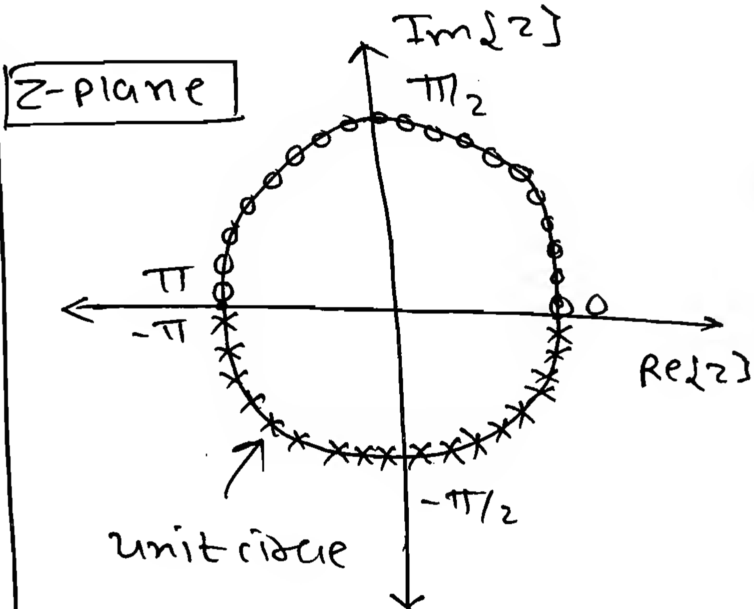
if $r = 1$

$$\Rightarrow X(z) = F.T. \{ x(n) \}$$

$$\Rightarrow \boxed{Z.T = D.T.F.T.}$$

L.T.	Z.T
$X(s) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-st} dt$	$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$.
$X(s) = F.T\{ x(t) \cdot e^{-\sigma t} \}$	$X(z) = F.T. \{ x(n) \cdot r^{-n} \}$
if $\sigma = 0 \Rightarrow s = j\omega$	if $r = 1$
L.T = C.T.F.T	Z.T = D.T.F.T.





$\Rightarrow$ L.T. calculated on $j\omega$ axis is C.T.F.T.

$\Rightarrow$ Z.T. calculated on unit circle is D.T.F.T.

Note:

⇒ +ve part of the 'jω' axis is corresponds to upper half of the unit circle. (ω varies from 0 to π).

⇒ −ve part of the 'jω' axis is corresponds to lower half of the unit circle (ω varies from 0 to −π).

⇒

$$x(t) = e^{st} \rightarrow \text{Cont}^n.$$

$$\downarrow t = nT_s$$

$$x(nT_s) = e^{snT_s} \longrightarrow \text{Discrete}$$

$$\Rightarrow \quad x[n] = \left(e^{sT_s}\right)^n \Leftrightarrow x[n] = z^n$$

$$\Downarrow$$

$$\boxed{z = e^{sT_s}}$$

⇒ The range of values of 'z' for which eq (1) is defined

$$\left[\sum_{n=-\infty}^{\infty} |x(n) z^{-n}| < \infty \right]$$ is R.O.C. of Z.T.